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⑧

FLUID MECHANICS

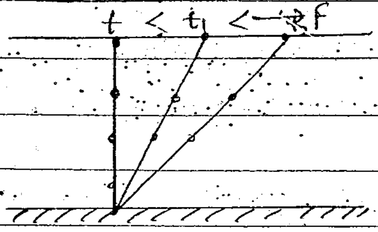
⑧ FLUID -MECHANIS

A fluid is a Substance which is capable of flowing under the action of shear force (however small the force may be), as long as shear force is there the fluid moves or deforms continuously.

e.g.- liquids, gases, vapour etc.

When the Shear force is removed, the fluid will never regain its original position. In case of solids if the shear force is within elastic limit, after removal of load it tries to regain its original shape. In fluid deformation rate is important than deformation.

** For a static fluid shear force = 0



$\boxed{\beta = \frac{1}{K}}$

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A fluid is said to be incompressible if its density remains Constant w.r.t pressure, liquids are generally incompressible and gases are Compressible.

If the mach Number (M) < 0.3, then the fluid flow can be treated as Incompressible.

** Bulk modulus increases with increase in pressure, this is because at higher pressure there will be greater resistance for the further Compression.

0 Isothermal Bulk Modulus of an ideal gas

$$PV = mRT$$

$$P = \frac{m}{V} RT$$

$$\therefore P = \rho RT$$

as $T = C$

$$\Rightarrow \frac{dP}{d\rho} = RT$$

as $K = \frac{\rho dP}{d\rho}$ $\Rightarrow K_T = \rho \times RT$

$$\Rightarrow \boxed{K_T = P} \checkmark$$

0 Adiabatic Bulk Modulus of an ideal gas

$$PV^{\gamma} = \text{Const.}$$

$$\rho = \frac{m}{V}$$

$$\Rightarrow V = m/\rho$$

$$\Rightarrow P\left(\frac{m}{\rho}\right)^{\gamma} = \text{Const.}$$

$$\Rightarrow \frac{P.m^{\gamma}}{\rho^{\gamma}} = \text{Constant}$$

$$\frac{P}{e^{\gamma}} = \frac{Const.}{m^{\gamma}}$$

$$\Rightarrow \frac{P}{e^{\gamma}} = Const.$$

$$\Rightarrow P = ce^{\gamma} \quad \text{—(i)}$$

$$\Rightarrow \frac{dP}{de} = c\gamma e^{\gamma-1}$$

as $K_a = e \times c\gamma e^{\gamma-1}$

$= c\gamma e^{\gamma}$

$\Rightarrow K_a = \gamma P$ (from (i))

Q1. The eqn of state for a liquid is $P = (3500e^{1/2} + 2500)\,N/m^2$, then find the bulk modulus at a pressure of $10^5 N/m^2$.

Ans. $P = (3500e^{1/2} + 2500)$

$$\Rightarrow \frac{dP}{de} = 3500 \times \frac{1}{2} \times e^{1/2-1} + 0$$

$$\Rightarrow \frac{dP}{de} = \frac{1}{2} \times (3500e^{-1/2})$$

$$\Rightarrow P - 2500 = 3500e^{1/2}$$

$$K = \frac{e\,dP}{de} = e \times \frac{1}{2} \times 3500 \times e^{-1/2}$$

$$\Rightarrow K = \frac{1}{2}(3500)e^{1/2}$$

$$\Rightarrow K = \frac{1}{2}(P - 2500)$$

at $P = 10^5\,N/m^2$

$$\Rightarrow K = 48750\,N/m^2$$

FLUID PROPERTIES

0 Density or mass density (ρ).

It is defined as the ratio of mass of the fluid to its volume, Its Unit is kg/m^3 and its dimensional formula is (ML^{-3}). Density depends on temp and pressure. Its an Absolute value.

ρ — Temp ↑ — ρ ↓
ρ — Press ↑ — ρ ↑

Density of water is $1000 kg/m^3$.

0 Specific Weight or Weight Density (w)

$$w = \frac{\text{Wt. of fluid}}{\text{Vol.}} \rightarrow \frac{N}{m^3} = ML^{-2}T^{-2}$$

It is defined as the ratio of weight of the fluid to its volume

$$w = \frac{mg}{vol} = \rho g = 1000 \times 9.81 = 9810 \ N/m^3.$$

w — ρ — Temp
w — ρ — Pressure
w — g (Location)

It is not an Absolute value.

** Density is an absolute quantity whereas specific quantity because it varies from place to place.

0 Specific Gravity (s)

It is defined as the ratio of density of fluid to the density of

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Standard fluid. It is dimensionless, for liquids the standard fluid is water and for gases the standard fluid is either H_2 or air at a given temp. and pressure.

The specific gravity of water is 1 and that of mercury is 13.6
Specific gravity can also be defined as, specific weight of fluid to specific weight of standard fluid.

$$S = \frac{\rho}{\rho_{\text{stand fluid}}} = \frac{\rho \times g}{\rho_{sf} \times g}$$

Compressibility (β)

It is reciprocal of Bulk modulus 'K', $\beta = \frac{1}{K}$

$$K = \frac{dP}{\frac{-dv}{v}} \Rightarrow \beta = \frac{1}{K}$$

$$\rho = \frac{m}{v}$$

$$\rho v = m \Rightarrow \rho v = \text{Constant}$$

On differentiation,

$$\rho dv + v d\rho = 0$$

$$\Rightarrow \frac{dv}{v} = -\frac{d\rho}{\rho}$$

$$K = \frac{dP}{\frac{d\rho}{\rho}} \Rightarrow K = \frac{\rho dP}{d\rho}$$

$$\Rightarrow \beta = \frac{d\rho}{\rho dp}$$

$\beta = 0 \Rightarrow$ incompressible,

$\Rightarrow d\rho = 0 \Rightarrow \rho = \text{Constant}$.

Q2. An increase in pressure by 2bar decreases the volume of liquid by 0.01% then find the bulk modulus.

(a). 2×10^5 N/m²

(b) 2×10^7 N/m²

(c) 2×10^9 N/m²

(d) 2×10^{11} N/m²

Ans. $K = \frac{+dP}{-\frac{dv}{V}}$

$dP = 2\times10^5$ N/m²

$\Rightarrow \frac{dP}{-\frac{dv}{V}} = \frac{2\times10^5}{\frac{0.01}{100}} = 2\times10^9$ N/m²

Q3. When the pressure on a given mass of liquid is increased from 3MPa to 3.5MPa the density of liquid increased from 500kg/m³ to 501kg/m³. find bulk modulus in MPa.

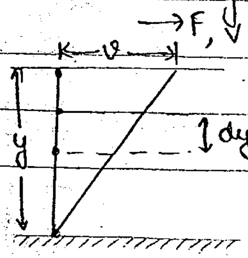
Ans. $dP = (3.5-3)$ MPa $= 5\times10^5$ Pa

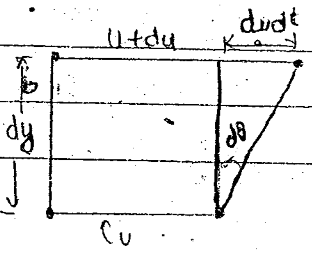
$\frac{d\rho}{\rho} = \frac{501-500}{500}$

$\Rightarrow K = \frac{\rho\, dP}{d\rho} = \frac{1}{1/500} \times 0.5$MPa $= 250$MPa

Viscosity — Viscosity नही तो fluid mechanics नही

It is the internal resistance offered by one layer of fluid by to the other adjacent layer.





$\tan d\theta = \frac{du\,dt}{dy}$

as $d\theta$ is small

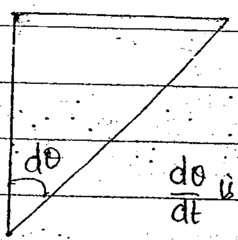
$\Rightarrow d\theta = \frac{du\,dt}{dy}$

$\Rightarrow \frac{d\theta}{dt} = \frac{du}{dy}$

$\tau = \frac{F}{A}$

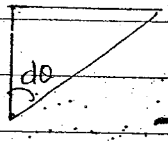
$\tau \propto \frac{d\theta}{dt} \Rightarrow \tau = \mu \frac{d\theta}{dt}$

$\Rightarrow \mu = \frac{\tau}{d\theta/dt}$



$\frac{d\theta}{dt}$ is more

$\Rightarrow$ flow is easy, Resistance is less μ is less.



$\frac{d\theta}{dt}$ = small

flow is not easy

μ is large, Resistance is large

μ is known as Absolute viscosity or dynamic viscosity or Coefficient of Viscosity.

$\tau = \mu \frac{d\theta}{dt} \Rightarrow \frac{d\theta}{dt} = \frac{du}{dy}$

$\Rightarrow \tau = \mu \frac{du}{dy}$

$\frac{d\theta}{dt}$ → Rate of angular deformation or rate of shear strain

$\frac{du}{dy}$ → Velocity gradient.

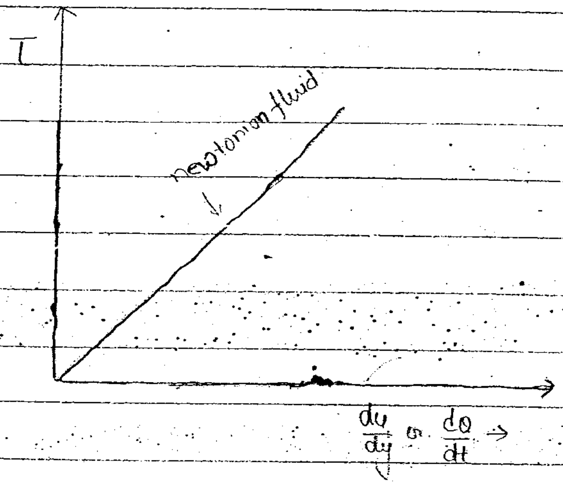
Newton's law of Viscosity

A fluid is said to be a newtonian fluid if shear stress is directly proportional to rate of shear strain or rate of angular deformation or velocity gradient. for a newtonian fluid

$$\tau = \mu \frac{du}{dy}$$

for a newtonian fluid μ is Constant.

Eg- Air, H_2O, Petrol, Kerosene, Diesel, Mercury etc.



Unit of Viscosity μ

$$\tau = \mu \frac{du}{dy}$$

$$\frac{N}{m^2} = \mu \times \frac{m}{s} \times \frac{1}{m}$$

$$\Rightarrow \mu = \frac{N\text{-}s}{m^2} = \frac{Kg\,m}{s^2} \times \frac{s}{m^2} = \frac{kg}{m\text{-}s}$$

$$\frac{N\text{-}s}{m^2} = Pa\text{-}s = \frac{Kg}{m\text{-}s} = \frac{M}{LT} = ML^{-1}T^{-1}$$

Unit of Viscosity in CGS system

Agitation - disturbance

(S.I) $\frac{kg}{m-s} = \frac{gm}{cm-s}$ (C.G.S) = poise

$\frac{1kg}{m-s} = \frac{10^3 gm}{10^2 cm-s}$

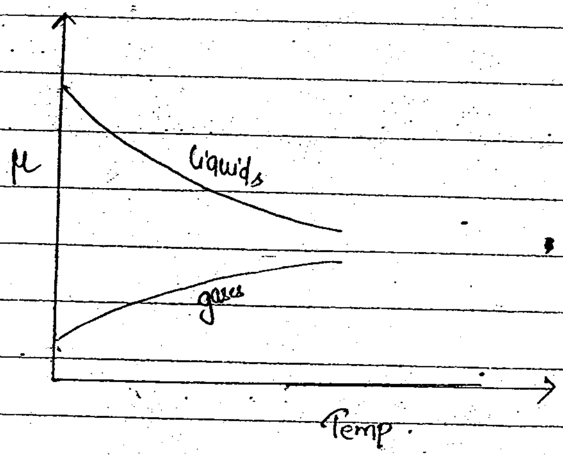
$\frac{1kg}{m-s} = 10$ poise

$\Rightarrow$ 1 poise = $\frac{0.1 kg}{m-s}$

Variation of viscosity with temp.

In case of liquids, intermolecular distance is small and hence cohesive forces are large, with rise in temp. cohesive forces decrease and the resistance to the flow also decreases.
∴ Viscosity of a liquid decreases with increase in temp

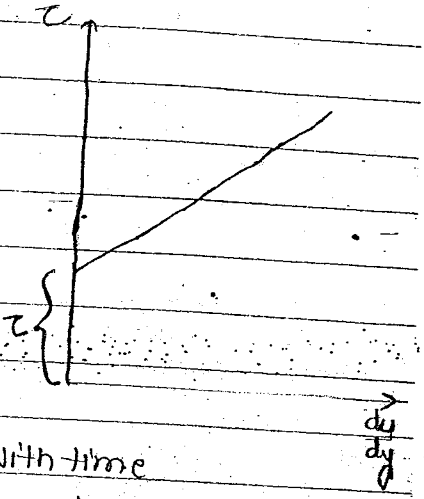
In case of gases the intermolecular distance are large and hence cohesive forces are negligible, with rise in temp. molecular agitation or disturbation increases and hence resistance to the flow also increases ∴ Viscosity of a gas increases with increase in temp.



$\frac{du}{dy} = 0$ (no flowabilily in acting as solid)

$\frac{du}{dy} > 0$ (flowability)

Case (iii) Bingham plastic or Ideal plastic



for bingham plastic fluid $B \neq 0$ and $n = 1$

Eg- tooth paste

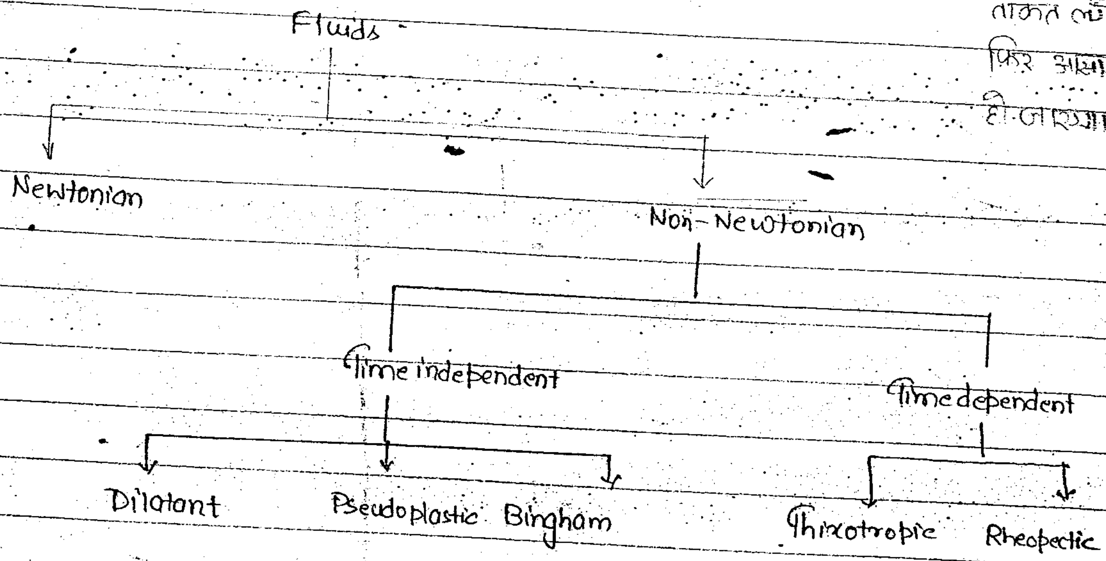
We have to cross threshold 'τ' to make it to flow.

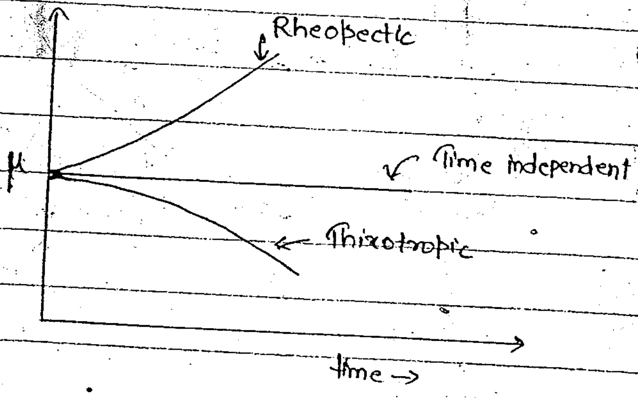
Certain fluids shows change in viscosity with time and these fluids are thixotropic & Rheopectic fluids.

Thixotropic fluids show decrease in viscosity with time eg- Paints, Lip-sticks etc

को हिलाने मे पहले बहुत ताकत लगेगा फिर आसान हो जायेगा

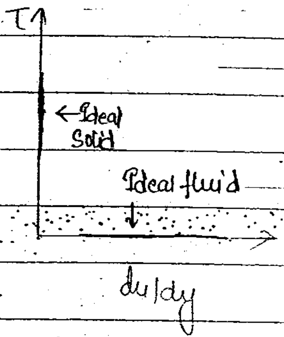
Rheopectic fluids shows increase in viscosity with time eg- Bentonitic solⁿ.





Ideal Fluids :-

An Ideal fluid is a fluid which is non-viscous and incompressible. The Concept of ideal fluid was introduced by mathematician for simplicity in analysis



Q4. A flat plate of 0.1 m² area is pulled at 30 cm/s relative to another plate located at a distance of 0.01 cm from it. The viscosity of the fluid is 0.001 $\frac{N\text{-}s}{m^2}$, then find the power required to maintain the velocity.

Ans. $P = F \times V$ $\mu = 0.001\ NS/m^2$

and $F = \frac{\mu A V}{y}$ $A = 0.1\,m^2$

$V = 0.3\,m/s$; $y = 0.01 \times 10^{-2}\,m$

$$= \frac{0.001 \times 0.1 \times 0.3}{0.01 \times 10^{-2}}$$

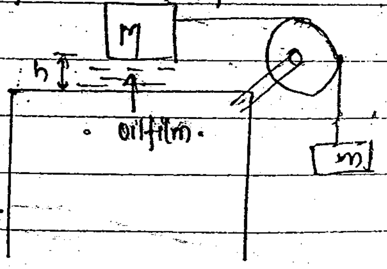
$F = 0.3N$

$\Rightarrow P = F \times V$

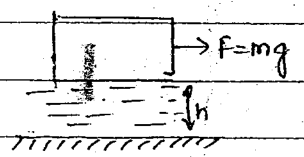
$= 0.3 \times 0.3$

$= 0.09\ W$

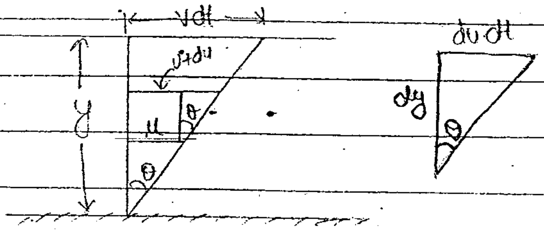
Q5. A block of mass M slides on a horizontal table on oil film of thickness 'h', the mass 'm' causes the movement, then find the expression for velocity if A is area of the block.



Ans.



Equation for a Linear Velocity profile



$$\tan\theta = \frac{vdt}{y}$$

$$\tan\theta = \frac{dudt}{dy} \Rightarrow \frac{vdt}{y} = \frac{dudt}{dy}$$

$$\Rightarrow \frac{V}{y} = \frac{du}{dy}$$

$$\Rightarrow \tau = \mu\frac{du}{dy}$$

$$\Rightarrow \tau = \frac{\mu v}{y}$$ — Valid for Linear Vel. profile

$$\tau = \frac{F}{A} = \frac{\mu V}{y}$$

$$\Rightarrow \boxed{f = \frac{\mu AV}{y}}$$ — Valid for Linear Velocity profile

** The Viscosity of Water at 20°c is 10^{-3} $\frac{N\text{-}s}{m^2}$
Water is 50-55 times more Viscous than air.

Kinematic Viscosity (ν)

Kinematics → not force in account
Dynamic → force taken into account.

In fluid mechanics the term μ/e occurs frequently and for Convenience this is termed as Kinematic Viscosity

$\boxed{\frac{\mu}{e} = \nu}$, It's Unit in m^2/s

In C.G.S system its Unit is cm^2/sec = Stoke

22/12/2011

Non-Newtonion Fluids

Fluids which donot obey newton's law of viscosity are known as non-newtonian fluids. The study of non-newtonian fluid is known as Rheology.

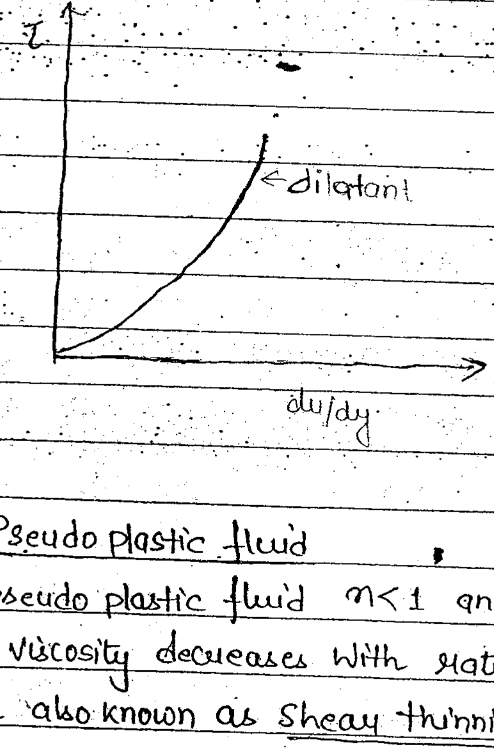
The general relationship b/w τ and $\frac{du}{dy}$ is

$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

Case(i) Dilatant fluid

for a dilatant fluid $n>1$ and $B=0$, for these fluids the apparent viscosity increases with rate of deformation and hence these fluids are also known as shear thickening fluids (i.e. shear stress increasing)

as Sugar in water, rice starch



$$\tau = A\left(\frac{du}{dy}\right)^n + B$$

$$\Rightarrow \tau = A\left(\frac{du}{dy}\right)^n$$

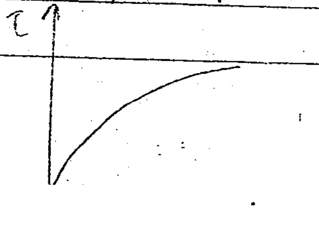
$$\Rightarrow \tau = A\left(\frac{du}{dy}\right)^{n-1} \cdot \left(\frac{du}{dy}\right)$$

$$\Rightarrow \tau = \mu_{app} \cdot \left(\frac{du}{dy}\right)$$

Case(ii) Pseudo plastic fluid

for a pseudo plastic fluid $n<1$ and $B=0$, for these fluids apparent viscosity decreases with rate of deformation these fluids are also known as Shear thinning fluids.

Eg:- blood, milk, Colloidal solns



$$F = \frac{\mu A V}{y}$$

$$\Rightarrow V = \frac{Fy}{\mu A}$$

$$\Rightarrow V = \frac{mgh}{\mu A}$$

Q6. The following stress strain rate relationship was obtained for a fluid then the fluid under consideration is

(a). Bingham

(b) Newtonian

(c). Dilatant

(d). Pseudoplastic

du/dy	0	2	3
τ	0	1.4	18

Ans. $\tau = A\left(\frac{du}{dy}\right)^n + B$

$0 = A(0) + B \Rightarrow B = 0$

$\tau = A\left(\frac{du}{dy}\right)^n \Rightarrow 1.4 = A(2)^n$

$\Rightarrow 18 = A(3^n)$

$\Rightarrow (1.5)^n = 12.85$

$\Rightarrow n > 1 \Rightarrow$ dilatant

Q7.

List 1	List II
A. Specific weight	(i) L/T^2
B. Density	(ii) F/L^3
C. Shear stress	(iii). F/L^2
D. Viscosity	(iv). FT/L^2
	(v). FT^2/L^4

Q8. A piston of 60mm diameter moves inside a cylinder of 60.1mm diameter. determine the percentage decrease in force necessary to move the piston When the lubricant is heated from 0°c to 120°c

$\mu_{0°c} = 0.0182\ Ns/m^2$

$\mu_{120°c} = 0.00206\ Ns/m^2$.

Ans.

$$\tau_{0°c} = 0.0182 \times \left(\frac{du}{dy}\right)$$

$$F_{0°c} = 0.0182 \times A \times \left(\frac{du}{dy}\right)$$

$$F_{120°c} = 0.00206 \times A \times \left(\frac{du}{dy}\right)$$

$$\% \text{ decrease in force} = \frac{0.0182\, A\left(\frac{du}{dy}\right) - 0.00206 \times A \times \left(\frac{du}{dy}\right)}{0.0182 \times A \times \left(\frac{du}{dy}\right)}$$

$$= \underline{\underline{88.68\%}}$$

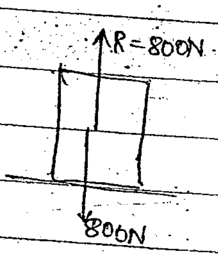
Q9 A skater Weighing 800N, skates at the rate of 15m/s on ice at 0°c the average skating area supporting him is 10cm² and the Coefficient of friction b/w skates and ice is 0.02 if there is actually a thin film of water b/w skates and ice then find its thickness take Viscosity of water as $10^{-3}\ N\text{-}s/m^2$.

Ans.

$F_f = f \times R$

$\Rightarrow F = 0.02 \times 800$

$= 16N$

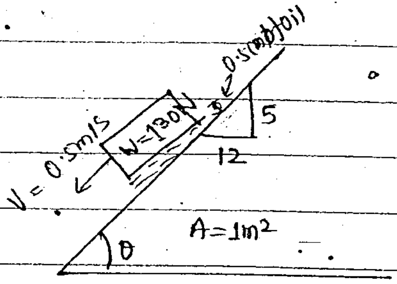


frictional force induces shear.

$$F = \frac{\mu A V}{y}$$

$$\Rightarrow 16 = \frac{10^{-3} \times 10 \times (10^{-2})^2 \times 15}{y} \Rightarrow y = 9.375 \times 10^{-7} m$$

Q10. Find the viscosity of oil from the fig. Shown.



Ans. $F_t = mg\sin\theta$

$= 130 \times \frac{5}{13} = 50N$

$\Rightarrow 50 = \mu \times \frac{0.5 \times 1}{0.005}$

$\Rightarrow 50 = \mu \times 100 \Rightarrow \mu = 0.5 \ \frac{N\text{-}s}{m^2}$

Q11. The velocity distribution for flow over a plate is given by $u = 0.5y - y^2$, Where y is distance above the plate, If the viscosity of the fluid is 0.9 N-s/m² then find the shear Stress at 0.2m above the plate.

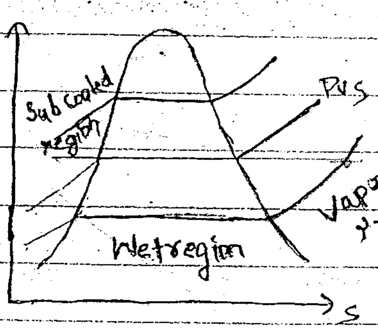
Ans. $u = 0.5y - y^2$

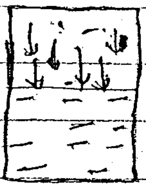
$\Rightarrow \frac{du}{dy} = 0.5 - 2y$

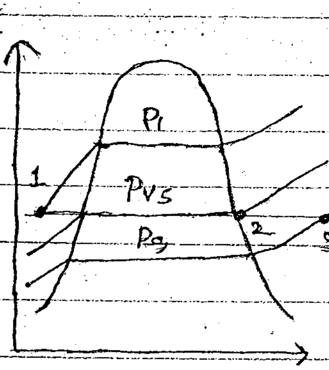
$\Rightarrow \left(\frac{du}{dy}\right)_{y=0.2} = 0.5 - 2\times 0.2 = 0.1 \Rightarrow \tau = 0.9 \times \left(\frac{du}{dy}\right)_{y=0.2}$

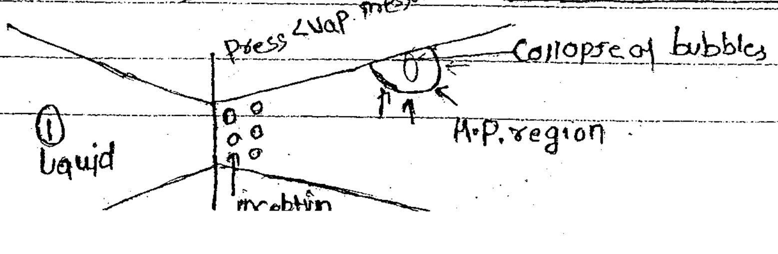
$= 0.9 \times 0.1 \Rightarrow \tau = 0.09 \ N/m^2$

Vapour Pressure









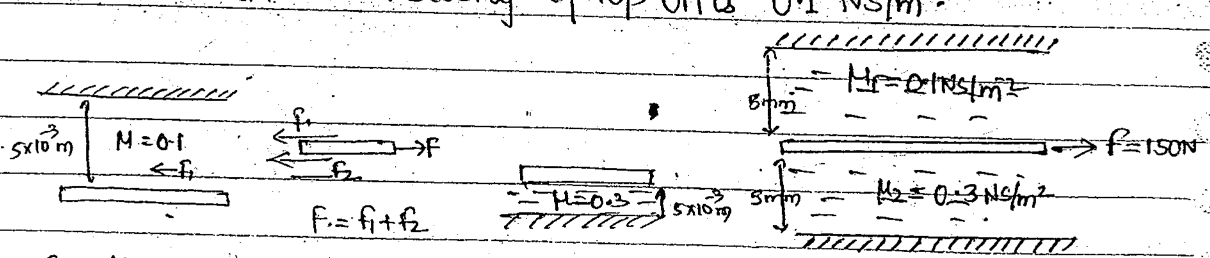
Flow through long pipes is taken as isothermal flow.

The liquid molecules at the surface escape due to more translational energy, if the container is closed, Once the saturation Cond^ns are reached, the no. of molecules escaping the surface is equal to no. of molecules rejoining the liquid surface on condensation. Under these conditions the pressure exerted by this vapour on the surface of this liquid is known as saturated vapour pressure. If the pressure of liquid at any point falls below vapour pressure, bubbles are formed (vaporisation occurs) and if these bubbles are carried to a region of high pressure these bubbles collapse and the surrounding fluid rushes to this region. This results in huge noise and the parts may be damaged and this phenomenon is known as Cavitation and hence, to avoid cavitation the pressure of liquid at any point must not be less than vapour pressure.

Vapour pressure depends on temperature, with increase in temp. as the molecular activity increases vapour pressure also increases. Highly volatile fluids like petrol have more vapour pressure. Mercury has low vapour pressure and because of this reason it is used in manometers.

Q12. In flow cond^ns given in fig. determine the velocity at which the central plate of area $5 m^2$ will move if a force of 150N is applied to it. Viscosity of two oils are in the ratio 1:3 and the viscosity of top oil is $0.1 \ Ns/m^2$.

Ans



$F = f_1 + f_2$

$$F = \frac{\mu A V}{y}$$

$$\Rightarrow F_2 = \frac{\mu_2 A V}{y_2}$$

$$\Rightarrow f_1 = \frac{\mu_1 A V}{y_1} = \frac{0.1 \times 5 \times V}{5 \times 10^{-3}}$$

$$\Rightarrow f_2 = \frac{0.3 \times 5 \times V}{10^{-3}}$$

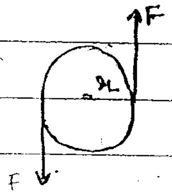
$$\Rightarrow \quad 150 = \frac{0.1 \times 5V}{5\times10^{-3}} + \frac{0.3\times5V}{5\times10^{-3}}$$

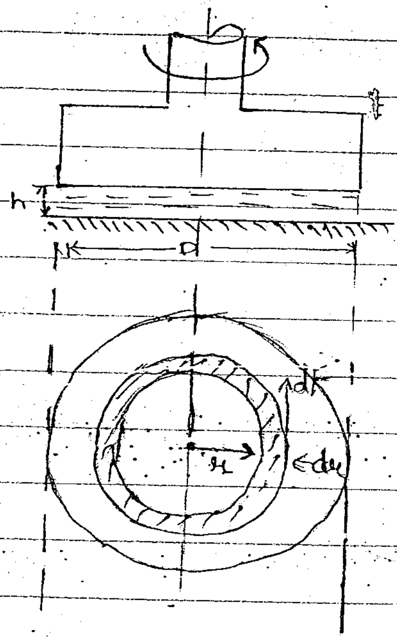
$$\Rightarrow \quad 150 = \frac{5V(0.3+0.1)}{5\times10^{-3}}$$

$$\Rightarrow \quad V = 0.375 \text{m/s}$$

Q3. A circular disc of diameter 'D' is kept at a height 'h' above a fixed surface by a layer of oil having a viscosity μ, determine the expression for torque on the disc.

Ans





$T = F \times r$

$F = \frac{\mu A V}{y}$

$V = r\omega$

$$dF = \frac{\mu (dA) V}{h}$$

$dT = dF \times r$

$$\Rightarrow dT = \frac{\mu\, dA \cdot V}{h} \times r$$

$$dT = \frac{\mu \times 2\pi r\, dr \times \omega r \times r}{h}$$

$$\Rightarrow T = \int_0^R \frac{2\pi\mu\omega r^3 dr}{h}$$

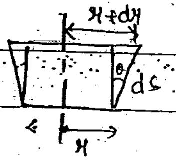
$$T = \frac{2\pi\mu\omega}{h}\left[\frac{r^4}{4}\right]_0^R$$

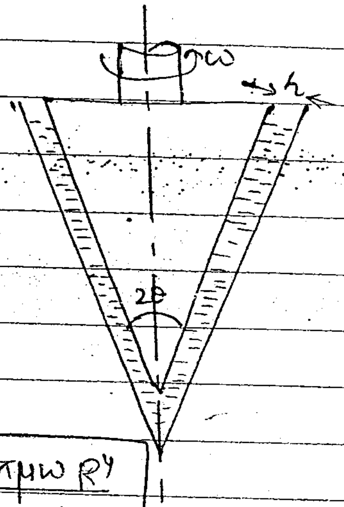
$$T = \frac{2\pi\mu\omega}{h}\left[R^4 - 0\right] \Rightarrow T = \frac{\pi\mu\omega R^4}{2h}$$

$$\Rightarrow T = \frac{\pi\mu\omega}{2h}\left(\frac{D}{2}\right)^4$$

Q4. A solid Cone of radius 'R' and Vertex angle 2θ is made to rotate at an angular velocity 'ω' in a Conical Cavity Containing oil with a viscosity μ. if 'h' is the gap b/w Cone and Cavity then find the torque to rotate the Cone.

Ans.





$$\sin\theta = \frac{dr}{ds} \Rightarrow ds = \frac{dr}{\cancel{d\theta}\,\sin\theta}$$

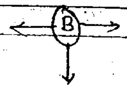
$$dA = 2\pi r\,ds$$

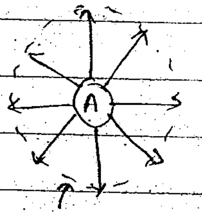
$$= \frac{2\pi r\,dr}{\sin\theta}$$

$$\boxed{T = \frac{2\pi\mu\omega R^4}{4y\sin\theta}}$$

23/12/2011

Surface Tension





Sphere of influence

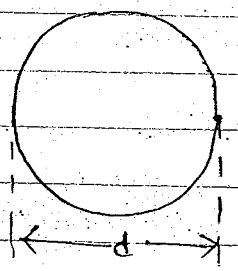
Consider molecule 'A' which is below the surface of the liquid. This molecule is surrounded by various corresponding molecules and hence under the influence of these forces (cohesive) it will be in equilibrium. Now, consider molecule 'B' which is on the surface of liquid, molecule is under the influence of net downward force and because of this there seems to be a membrane formed at the surface which can resist small tensile loads, this phenomenon is known as surface tension. Surface tension is a line-force and it is expressed as force per Unit length drawn on the surface and it acts normal to the line in the plane of surface.

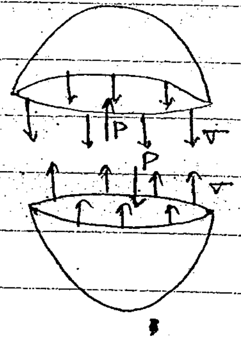
The Unit of surface tension is N/m and it's dimensional formula is MT^{-2}
The surface tension for Water-air interface at 20°C is 0.0736 N/m.
Surface tension is basically due to Unbalanced Cohesive force and with rise in temp. Cohesive forces decrease and hence Surface tension also decreases.
At critical point surface tension is zero because at critical point liquid vapour interface Vanishes.

Surfactants are used While Washing clothes to reduce Surface tension so that Water Can penetrate easily and Can remove dust.

Liquid droplets are spherical in shape due to surface tension because sphere has minimum surface area for a given volume.

Pressure inside a liquid drop in excess of atmospheric pressure





$P = \frac{F_P}{A} \Rightarrow F_P = P \times \frac{\pi}{4} d^2$

$\sigma = \frac{F_s}{L} \Rightarrow F_s = \sigma L$

$\Rightarrow F_s = \sigma \pi d$

for equilibrium,

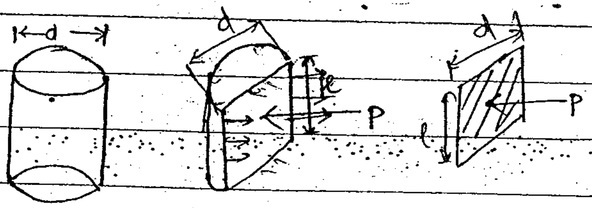
$F_P = F_s$

$\Rightarrow P \times \frac{\pi}{4} d^2 = \sigma \pi d$

$\Rightarrow \boxed{P = \frac{4\sigma}{d}}$

** In case of bubble there are two surfaces and hence P is equal to $\frac{8\sigma}{d}$ $\Rightarrow P = \frac{8\sigma}{d}$

Pressure inside a liquid jet in excess of atmospheric pressure,

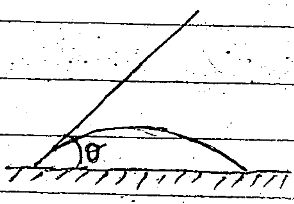


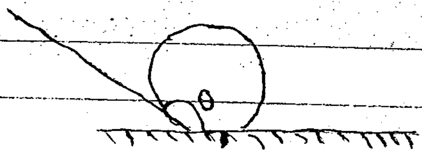
$$\sigma = \frac{f_s}{l} \Rightarrow f_s = \sigma \times L$$
$$\Rightarrow f_s = \sigma \times (l + l)$$
$$\Rightarrow f_s = 2\sigma l$$

$$P = \frac{F_p}{A} \Rightarrow F_p = PA$$
$$\Rightarrow F_p = PdL$$

for eqlb^m, $F_p = F_s$
$$\Rightarrow PdL = 2\sigma L$$
$$\boxed{P = \frac{2\sigma}{d}}$$

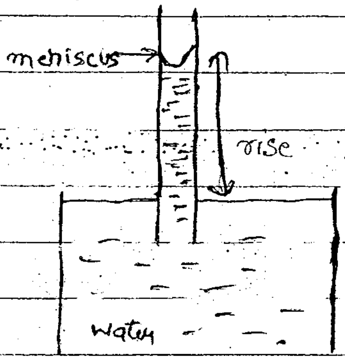
Capillarity



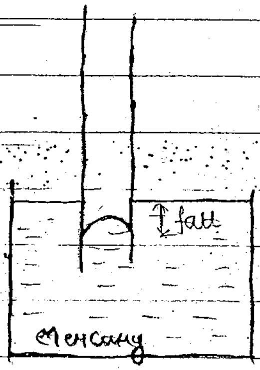


Adhesion is more	Cohesion is more
Wetting liquids	Non-Wetting liquids
$\theta < 90^\circ$	$\theta > 90^\circ$

The rise or fall of a liquid when a glass tube is immersed in it is known as Capillarity. The capillary rise is due to Adhesion (Eg-Water) and capillary fall is due to Cohesion (Eg- Mercury). Thus, capillarity is due to both adhesion and Cohesion



Adhesion more

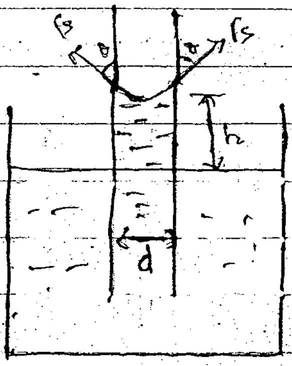


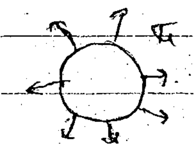
Cohesion more

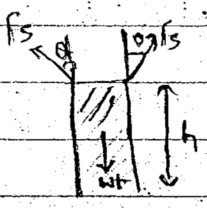
Expression for Capillary rise or fall

Surface tension at interface of liquid and tube

All over the circumference







$$w = \frac{Wt}{Vol} \Rightarrow Wt = w \times Vol$$

$$= w \times \frac{\pi d^2}{4} \times h$$

$$\sigma_s = \frac{Fs}{l} \Rightarrow Fs = \sigma l$$

$$\Rightarrow Fs = \sigma \pi d$$

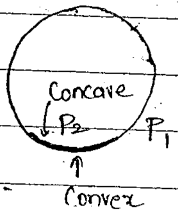
for eqlb^m,

Wt = Vertical Component of Fs

$$\Rightarrow w \times \frac{\pi}{4} d^2 \times h = \sigma \times \pi \times d \cos\theta$$

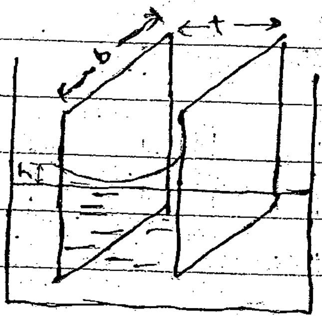
$$\Rightarrow \boxed{h = \frac{4\sigma\cos\theta}{wd}}$$

**
- The angle of Contact for pure Water with clean glass tube is 0°.
- The angle of Contact of Water with glass tube is about 29°.
- The angle of Contact of mercury with glass tube is 132°.
-



$P_2 > P_1$

Capillary rise b/w two parallel plates



$W = \frac{Wt}{Vol}$

$\Rightarrow Wt = \omega \times Vol$

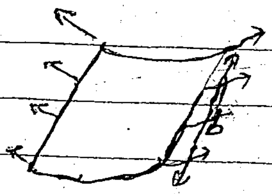
$\sigma = \frac{Fs}{L}$

$fs = \sigma \times L$

$= \sigma \times 2b$

$\Rightarrow$ Vertical Comp. $= f_s \cos\theta$

$= 2\sigma b \cos\theta$



for eqlb^m

$2\sigma b\cos\theta = wbth$

$$\Rightarrow \boxed{h = \frac{2\sigma\cos\theta}{wt}}$$

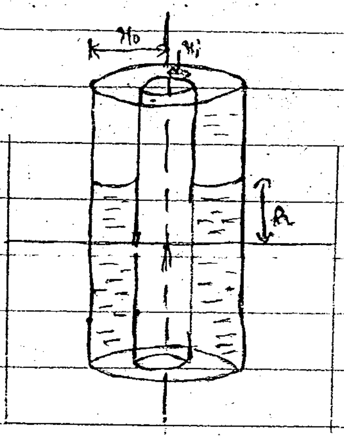
** If the height of the capillary tube is insufficient to the possible rise of the liquid, as there is no adhesion the liquid cannot rise beyond the length.

Capillary rise in the annulus of two coaxial ~~turbine~~ tubes

$$\omega = \frac{Wt}{Vol.}$$

$$Wt = \omega \times Vol.$$

$$\Rightarrow Wt = \omega\pi(r_0^2 - r_i^2)\times h$$



$$\sigma = \frac{F_s}{L}$$

$$\Rightarrow F_s = \sigma L$$

$$\Rightarrow F_s = \sigma\left[2\pi r_i + 2\pi r_o\right]$$

$\Rightarrow$ Vertical component $= F_s\cos\theta = 2\pi\sigma(r_0 + r_i)\cos\theta$.

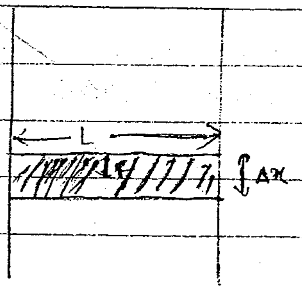
Wt = Vertical component of F_s.

$$\Rightarrow \omega\pi(r_0^2 - r_i^2)\times h = 2\pi\sigma(r_0 + r_i)\cos\theta$$

$$\Rightarrow \omega(r_0 - r_i)\,h = 2\sigma\cos\theta$$

$$\Rightarrow \boxed{h = \frac{2\sigma\cos\theta}{\omega(r_0 - r_i)}}$$

Work done in Stretching a surface



$$F = \sigma \times L$$

$$W = F\times\Delta x \Rightarrow Work = \sigma L \times \Delta x$$

$$\Rightarrow \boxed{Work = \sigma\left[\text{increase in Surface Area}\right]}$$

Q15. A small drop of water at 20°c has a dia. of 0.05 mm. If the pressure within the droplet is 0.6kpa higher than atmospheric pressure then find the surface tension.

Ans. $P = \frac{4\sigma}{d}$

$\Rightarrow \quad 0.6\times10^{3} = \frac{4\times\sigma}{0.05\times10^{-3}}$

$\Rightarrow \quad \sigma = 7.5\times10^{-3}\,N/m$

Q16. If the dia. of tube is 1mm then the capillary rise is 3cm, What will be the capillary rise when the diameter is changed to 0.2mm.

Ans $h = \frac{4\sigma\cos\theta}{wd}$

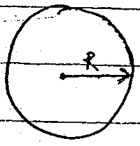
$\Rightarrow h \propto \frac{1}{d} \Rightarrow \frac{h_2}{h_1} = \frac{d_2}{d_1}$

$\Rightarrow h_1 = 3cm \Rightarrow d_1 = 1mm, \; d_2 = 0.2mm$

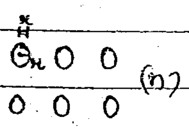
$\Rightarrow \quad h_2 = 15cm.$

Q17. A spherical water drop of radius 'R' splits up in air into 'n' smaller drops of equal size then find the work done in splitting the drop.

Ans Since volume remaining conserved



$\Rightarrow \frac{4}{3}\pi R^3 = n\times\frac{4}{3}r^3.$



$\Rightarrow R^3 = nr^3 \quad \Rightarrow \quad r = \frac{R}{n^{1/3}}$

Work = σ [Increase in Surface area]

$$\Rightarrow \text{Work} = \sigma\left[4\pi r^2 n - 4\pi R^2\right]$$

$$= \sigma \times 4\pi\left[n r^2 - R^2\right].$$

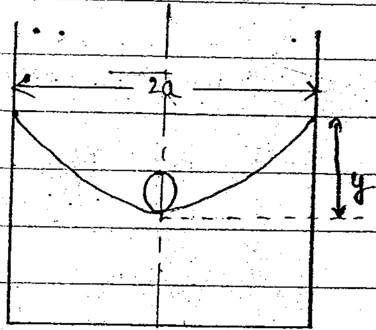
$$= \sigma \times 4\pi \times \left[\frac{R^2 . n}{n^{2/3}} - R^2\right]$$

$$= 4\pi\sigma R^2\left[n^{1/3} - 1\right]$$

$$= \underline{\underline{4\pi (n^{1/3}-1) R^2 \times \sigma}}$$

Q18. A container of width '2a' is filled with a liquid a thin wire of Weight λ per unit length is placed at the centre of the liquid surface as shown in fig. As a result the liquid surface is depressed by a distance 'y' ($y << a$) then find the surface tension of liquid.

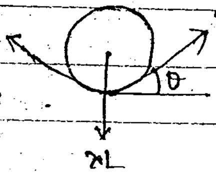
Ans. $Wt = \lambda L$

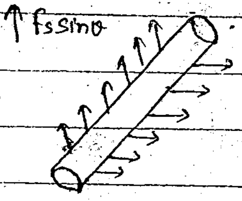


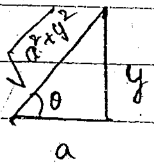
$\sigma = \frac{F_s}{L} \Rightarrow F_s = \sigma L$

both side length is providing adhesion and so surface tension

$\Rightarrow F_s = \sigma (l + l) = \sigma \times 2L$







$\sin\theta = \frac{y}{\sqrt{y^2+a^2}}$ as ($y << a$)

$\Rightarrow \sin\theta = \frac{y}{a}$

for eqlb^n

$f_s \sin\theta = 2\sigma l \sin\theta$

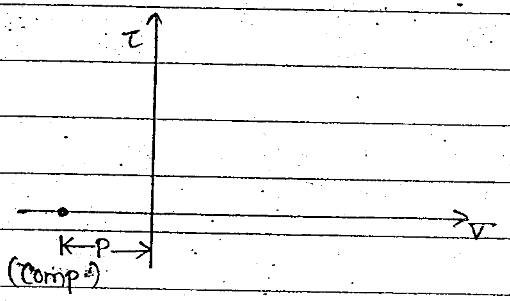
$\Rightarrow \lambda l = 2\sigma l \sin\theta \Rightarrow \lambda \cancel{l} = 2\sigma \cancel{l} \times y/a$

$\Rightarrow \sigma = \frac{\lambda a}{\underline{\underline{2y}}}$ Ans.

PRESSURE MEASUREMENT

MANOMETRY

Pressure is defined as normal force exerted by the fluid per unit area. It is compressive in nature. Its unit is N/m^2 or pascal. Pressure is also expressed in bar for comparing it with atmospheric pressure. For a static fluid as there is no shear stress only normal forces are present. therefore, mohr's circle is a point as shown in fig.



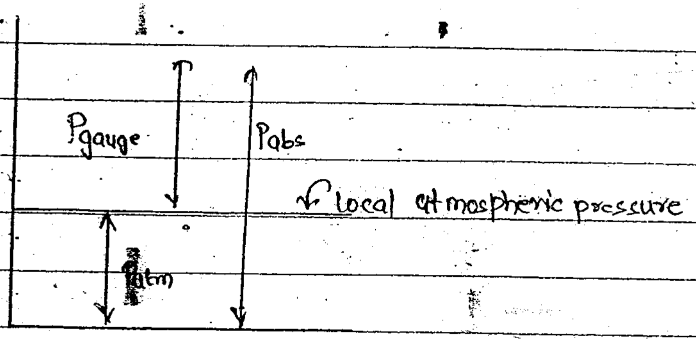
Mohr's circle (Static fluid)

Atmospheric pressure :- The pressure exerted by environmental mass is known as atmospheric pressure. It is measured by barometer. Standard value at 15°c. and $\rho = 1.15\ Kg/m^3$.

Gauge pressure :- Pressures measured relative to atmospheric pressure (local) is known as gauge pressure.

Absolute pressure :- Pressure measured w.r.t zero pressure is known as absolute pressure.

$$P_{abs} = P_{gauge} + P_{atm}$$



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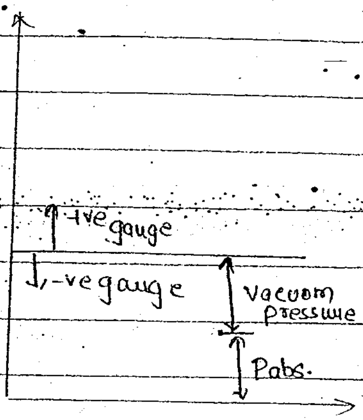
for finding Abs. pressure we take local atmospheric pressure as reference.

Vacuum Pressure

The pressure less than atmospheric pressure is known as vacuum pressure.

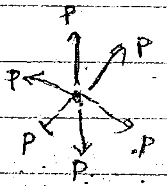
** There can be +ve gauge pressure and -ve gauge pressure but there cannot be -ve absolute pressure.



Pabs = Patm - Vacuum pr.

Pascal's Law

Pressure at a point in a static fluid is equal in all dirxns i.e. if pressure is applied at a point in a static fluid it is transmitted equally in all dirxns.



Same in all dirxns.
∴ Pressure is a scalar quantity

Applications

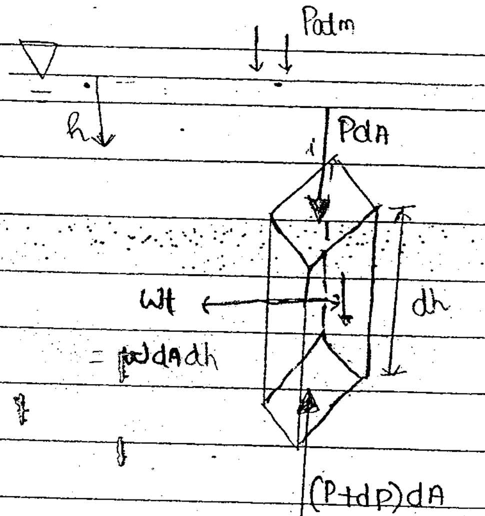
(a). Hydraulic lift
(b). Hydraulic brakes

Pascal's law can also be applied for flowing fluids if the fluid is ideal. As for an ideal fluid shear stress $(\tau = 0)$.

Hydrostatic Law

The variation of pressure in vertical dirn is proportional to specific weight.

A surface is called a free surface when all external forces are removed from the surface except the atmospheric pressure.



w (specific weight) $= \frac{Wt}{Vol.}$

$\Rightarrow \quad Wt = w \times Vol.$

$Wt = w\,dA\,dh$

At free surface.

$h = 0 \Rightarrow P = P_{atm}$

for equilibrium,

$$P\,dA + w\,dA\,dh = (P+dP)\,dA$$

$$\Rightarrow P + w\,dh = P + dP$$

$$\Rightarrow w\,dh = dP$$

$$\Rightarrow \boxed{\frac{dP}{dh} = w} \leftarrow \text{hydrostatic law.}$$

for gauge pressures, as atmospheric pressure is a reference line ∴ it is treated as zero.

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Pressure at any depth 'h' :-

$$\frac{dP}{dh} = w \Rightarrow dp = wdh$$

$$\Rightarrow P = wh + c$$

at free Surface, $h=0 \Rightarrow P = P_{atm}$

$$\Rightarrow P_{atm} = w(0)+c$$

$$\Rightarrow C = P_{atm}$$

$$\Rightarrow P = wh + P_{atm}$$

If P is gauge pressure, $\Rightarrow P_{atm} = 0$

$$\Rightarrow P = wh$$

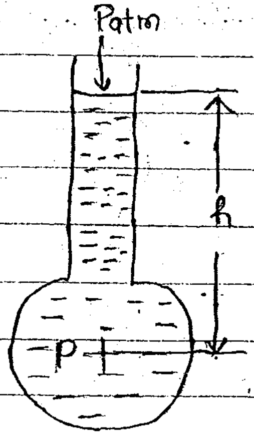
$$P = \rho gh \Rightarrow \frac{P}{\rho g} = h$$

$$\Rightarrow \boxed{h = \frac{P}{w}}$$ ← pressure head

** Pressure increases with depth and with height pressure decreases ∴ if 'h' is taken in vertically upward dirxn $\frac{dP}{dh} = -w$

Piezometer

Piezometer is a device which is opened to both ends one end connected to the point where pressure is to be found and the other end to atmospheric air. These are not suitable for very high pressure and they are also not suitable for gas pressures, ∴ piezometer are used for finding out moderate liquid pressures



$0 + \rho gh = P$

** In order to nullify the effect of capillarity the diameter of the tube must be greater than 1cm.

Manometry —

The technique of finding the pressure using hydrostatic law is known as manometry.

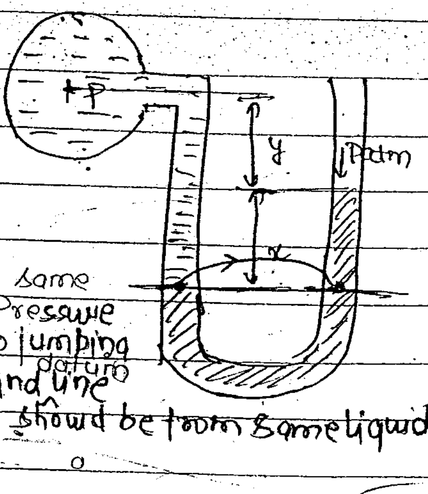
Manometer
- Simple
  - measure press. at a point
- Differential
  - measure press diff b/w two points.

Simple U-tube Manometers

There are two methods of finding pressure

(i). Jumping of fluid technique the fluid must be continuous

$$P+\rho g(x+y) - \rho_m g x = 0$$
$$P = \rho_m g x - \rho g(x+y)$$



same Pressure so jumping and datum line should be from same liquid

(ii). Datum line Technique

$$P_A = P_B$$
$$P+\rho g(x+y) = P_A$$
$$0+\rho_m g x = P_B$$
$$\Rightarrow P+\rho g(x+y) = \rho_m g x$$
$$P = \rho_m g x - \rho g(x+y)$$

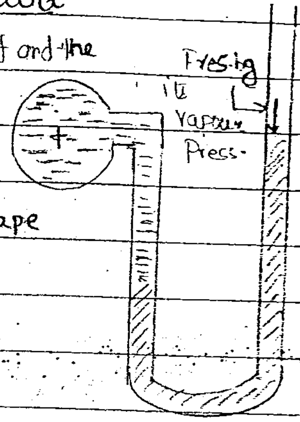
Reasons for using mercury as manometric fluid

higher ↑ 0. Mercury has low vapour pressure ↑ might exert press on itself and the calculation will be wrong. and more volatile may escape

0 It has high density (low longitude reqd).

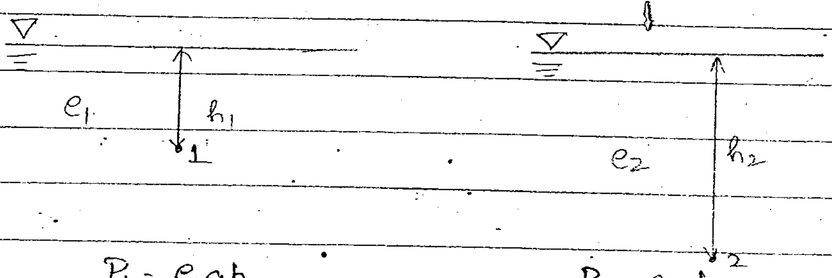
lower 0 - It is immiscible with other fluids.

Priority order



Many fluids are of high density but they don't have low vapour pressure.

Conversion of One fluid Column into another fluid Column



$P_1 = \rho_1 g h_1$ $P_2 = \rho_2 g h_2$

$\Rightarrow P_1 = P_2$

$\Rightarrow \rho_1 g h_1 = \rho_2 g h_2$

$\Rightarrow \rho_1 h_1 = \rho_2 h_2$

and $\frac{\rho_1}{\rho_{H_2O}} h_1 = \frac{\rho_2}{\rho_{H_2O}} h_2 \Rightarrow S_1 h_1 = S_2 h_2$

Q19. Pressures have been observed at 4 diff. points in diff. Units as follows.

(A) 150 KPa (B) 1800 millibar

(C) 20 m of Water (D) 1240 mm of mercury

Then arrange the points in the decreasing order & magnitude of pressure.

Ans. (A) 150 kpa — $150 \times 10^3 N/m^2$

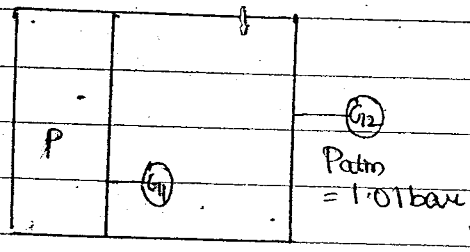
(B) 1800 millibar — $1800 \times 10^{-3} \times 10^5 N/m^2 = 180 \times 10^3 N/m^2$

(C) 20 m of water — $P = \rho gh = 10^3 \times 9.81 \times 20 = 196.2 \times 10^3 N/m^2$

(D) 1240 mm of Hg — $P = \rho gh = 13.6 \times 10^3 \times 9.81 \times \frac{1240}{1000} = 165.4 \times 10^3 N/m^2$

then C, B, D, A

Q20 The pressure gauges G_1 & G_{12} installed on a System shows pressure of $P_{G_1} = 5$ bar and $P_{G_{12}} = 1$ bar then find the Unknown pressure 'P' in absolute scale.



Ans for G_2 Patm. will be local Pressure

so $P_{1_{abs}} = 1 + 1.01$ bar

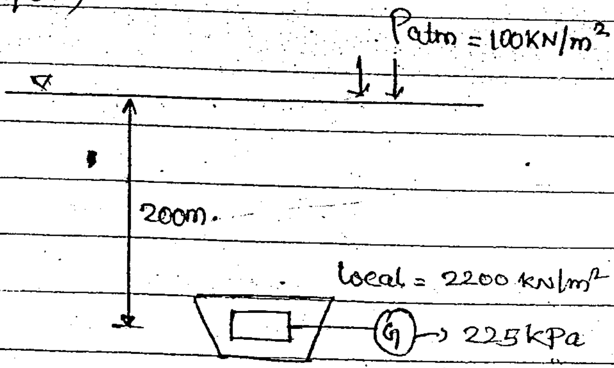
$= 2.01$ bar

for G_1, the local pressure will be

$P_{2_{abs}} = 5 + 2.01$

$= 7.01$ bar Ans

Q21 A diver descends 200 m in a sea ($\rho = 1050 kg/m^3$) to a ship wherein a Container is found with a pressure gauge reading of 225 kpa. Taking the pressure at Sea Surface to be atmospheric (100kpa). Then find the absolute pressure in a Container. (Take $g = 10 m/s^2$)



Ans Press at a depth 200m

$= \rho gh$ (gauge)

$1050 \times 10 \times 200$

$= 2100 \times 10^3 N/m^2$

$2100 KN/m^2$

Absolute press. at 200m depth

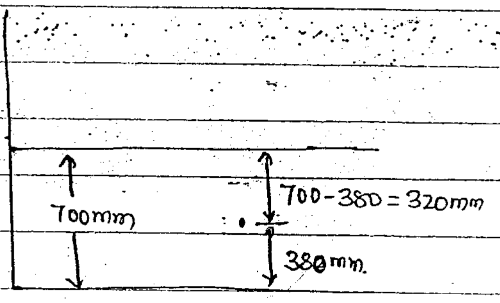
$= 2100 + 100 = 2200 kN/m^2$

$\Rightarrow P_{abs} = 225 + 2200 = 2425 KN/m^2$

Q22. The standard atmospheric pressure is 762 mm of Hg. At a specific location the local atmospheric pressure is 700 mm of Hg, at this place What does an absolute pressure of 380 mm of mercury Corresponds to

✓(a). 320 mm of Hg vaccum (b) 62 mm of Hg vacuum
(c). 382 mm of Hg vaccum (d) 300 mm of Hg vacuum

Ans



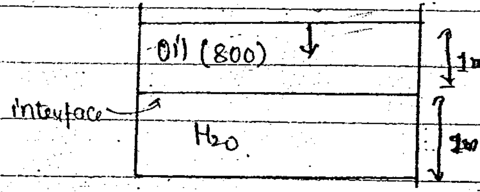
$\Rightarrow P_{abs} = 320$ mm Hg

Q23. An open tank contains water to a depth of 2m and oil over it to a depth of 1m, the density of oil is 800 kg/m³. Then find the pressure at the interface of two fluid layers.

Ans In fluid mechanics if nothing is given we measure gauge pressure

$0 + 800 \times 9.81 \times 1 = P_{interface}$

$\Rightarrow P_{int} = 7848\ N/m^2$.

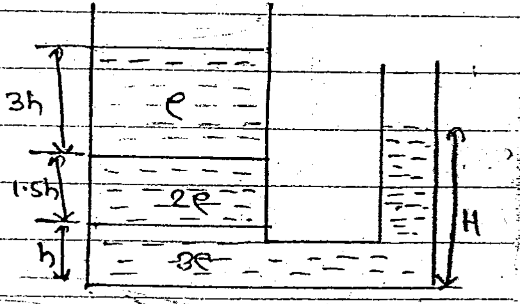


Q24. ~~Two~~ 3 Immiscible liquids of density $\rho, 2\rho$ and 3ρ are kept in a Jar and the piezometer fitted to the bottom of the Jar as shown in fig then find H/h.

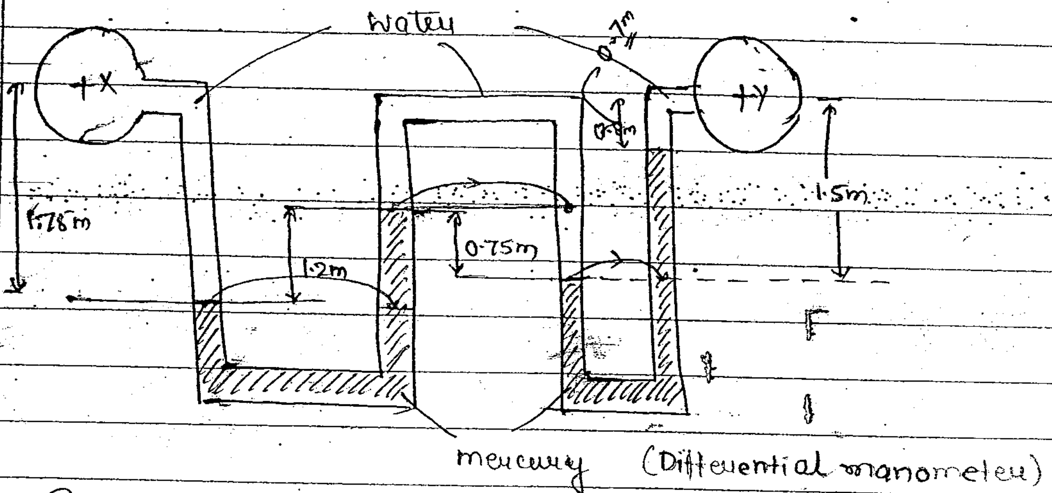
Ans $0 + \rho g(3h) + 2\rho g\,(1.5h) + 3\rho gh - 3\rho gH = 0$

$\Rightarrow 9\rho gh - 3\rho gH = 0$

$\Rightarrow \dfrac{H}{h} = 3$



Q25 Two U-tube manometers are connected in series as shown in fig. Then find the pressure difference by X & Y in kpa.



Ans. This manometer is known as multi U-tube manometers and are used for finding out very high pressures.

$\Rightarrow P_x + 10^3 \times 9.81 \times 1.76 - 13.6 \times 10^3 \times 9.81 \times 1.12 + 10^3 \times 9.81 \times 0.75$
$- 13.6 \times 10^3 \times 9.81 \times 0.8 - 10^3 \times 9.81(1.5-0.8) = P_y$

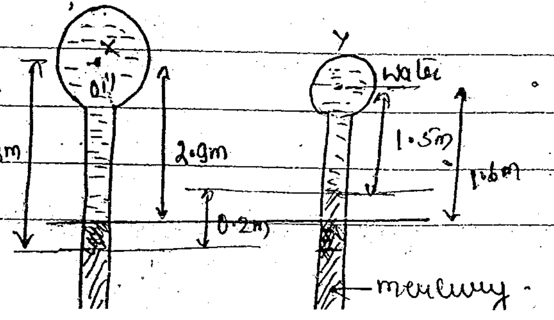
$\Rightarrow P_x - P_y = 238.5 \times 10^3 \text{ N/m}^2$

$\Rightarrow P_x - P_y = 238.5 \text{ KN/m}^2$

Q26 Two pipe lines one with oil of density 900 kg/m³ and other with water are connected to a manometer as shown in fig. by what amount the pressure in the water pipe should be increased without changing oil pressure so that mercury level in both limbs of manometer become equal. Take density of Hg as 13550 kg/m³.

Ans. Let the pressure be changed by Pe.

$\Rightarrow P_x + 900 \times 9.81 \times 3$
$- 13550 \times 0.2 \times 9.81$
$- 1000 \times 9.81 \times 1.5 = P_y$



$\Rightarrow P_x + 900 \times 2.9 \times 9.81 - 1.6 \times 9.81 \times 1000 = P_y + P_c$

$\Rightarrow P_x + 9908.1 = P_y + P_c$

$\Rightarrow 14813.1 + 9908.1 + \cancel{P_y} = \cancel{P_y} + P_c$

$\Rightarrow P_c = 24721.2 \text{ N/m}^2$

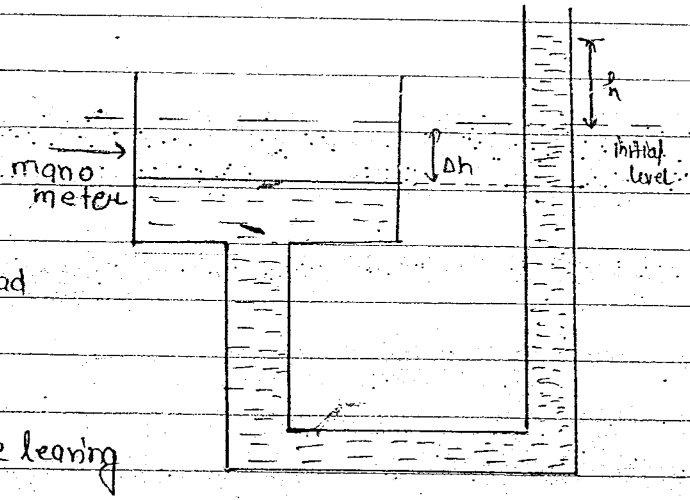
$= \underline{\underline{24.721 \text{ KN/m}^2}}$ Ans

Q27. The cross-sectional area of 1 limb of a U-tube manometer is made 500 times larger than the other so that the pressure diff. b/w the two limbs can be determined by measuring 'h' on one limb of manometer then find the percentage error w.r.t this pressure measurement.

Ans Actual pressure head

$= h + \Delta h$

Pressure calculated by using only 'h'.



$\therefore$ diff in both pressure head

$= h + \Delta h - h = \Delta h$

as volume entering = Volume leaving

$\Rightarrow A\Delta h = ah$

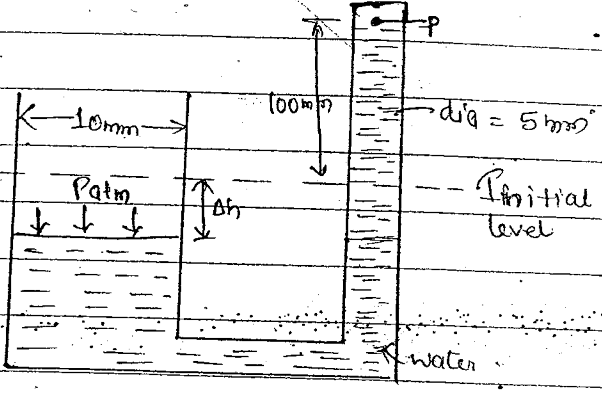
$\Rightarrow \dfrac{\Delta h}{h} = \dfrac{a}{A} = \dfrac{1}{500}$

$\therefore$ Error $= \Delta h$

$\Rightarrow \% \text{ Error} = \dfrac{\Delta h}{h} \times 100 = \underline{0.2\%}$ Ans.

Q28. A U-tube manometer as shown in fig. has water as a manometric fluid. When an unknown pressure 'p' acts at 5mm diameter limb, water rises in the limb by $\overset{100}{\cancel{10}}$mm from initial level.

If other end is opened to atmosphere, then find P−Pa in N/m².



Ans : $\Delta h \times \cancel{\frac{\pi}{4}} \times (10\,mm)^2 = 100 \times \cancel{\frac{\pi}{4}} \times (5)^2$

$\Rightarrow \Delta h = 25\,mm$

$\Rightarrow \cancel{(100+25)\times 10^{-3}} \times 1000$

$= \cancel{125\,N/m^2}$

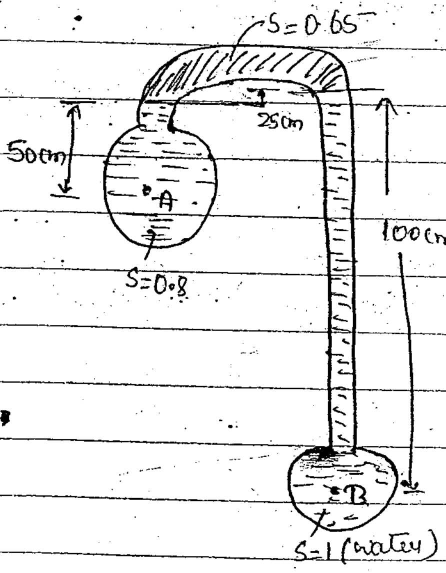
$P_a \cancel{+} - (125)\times 10^{-3} \times 1000 \times 9.81 = P$

$\Rightarrow P - P_a = -125 \times 9.81$

$= -1226.25\,N/m^2$ Ans.

27/12/2011

*** Q29. Find the pressure difference b/w point B and A as shown in fig in cm of water



Ans Press. diff b/w B and A

i.e: $P_B - P_A$

$h_1 S_1 = h_2 S_2$

$\Rightarrow 50 \times 0.8 = h_w \times 1$

$\Rightarrow h_w = 50 \times 0.8$

$\Rightarrow h_w = 40\,cm$

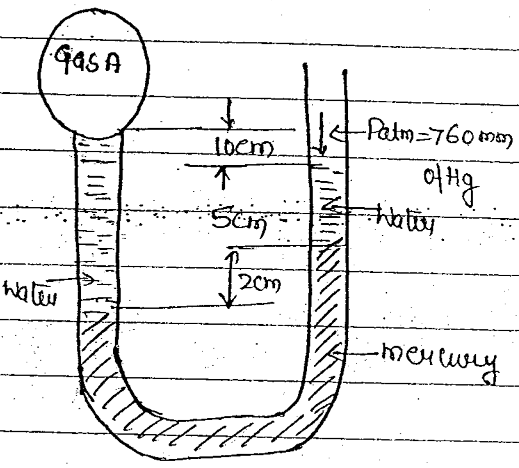
$25 \times 0.65 = h_w \times 1$

$\Rightarrow P_A - 50 \times 0.8 - 25 \times 0.65 + 100 = P_B$

$\Rightarrow P_B - P_A = 43.75$ cm of water

Q30. Refer to the figure and find absolute pressure of gas A in mm of Hg.

Ans.



$$P_A + \frac{\overset{170}{\cancel{200}}}{13.6} \overset{-}{\cancel{+}} 20 \overset{-}{\cancel{+}} \frac{150}{13.6} = 0$$

$$\Rightarrow P_A + 18.53 = 0$$

$$\Rightarrow P_A = -18.53 \text{ mm}$$

$$\Rightarrow P_{A\,abs} = \cancel{778.53\text{ mm}}\ 760 - 18.53$$

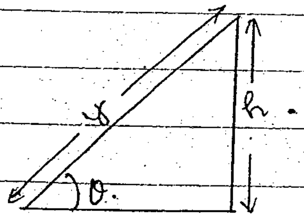
$$= 741.47 \text{ mm}$$

$$\Rightarrow P_A + \frac{170}{13.6} - 20 - \frac{50}{13.6} = 0$$

$$\Rightarrow P_A\text{(gauge)} = 11.17 \text{ mm}$$

$$\Rightarrow P_A\text{(abs)} = 760 + 11.17 = 771.17 \text{ mm of Hg}$$ Ans

~~Q31~~. Sensitivity of Inclined Manometer



$$\sin\theta = \frac{h}{y}$$

$$\Rightarrow y = h \times \left(\frac{1}{\sin\theta}\right)$$

In comparison to 'h', y is increased by $\frac{1}{\sin\theta}$

$$\therefore \boxed{\text{Sensitivity} = \frac{1}{\sin\theta}}$$

*****

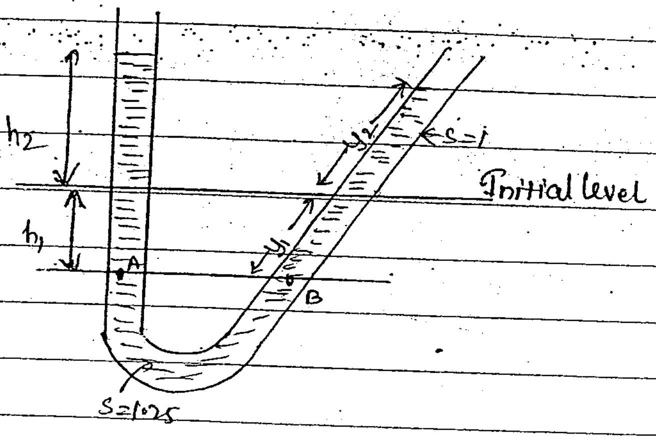
Q31 A manometer is made of tube of uniform X-sectional area with 0.5 cm², with one limb vertical and the other limb inclined at 30° to horizontal both limbs are open to atmosphere, initially it is partially filled with a liquid of specific gravity 1.25, if now an additional volume of 7.5 cm³ of water is added to inclined tube Calculate the rise of liquid in vertical tube from initial level.

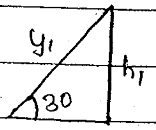
Ans. Area x length = Vol

$\Rightarrow 0.5 \times (y_1 + y_2) = 7.5$

$\Rightarrow y_1 + y_2 = 15$

$\Rightarrow y_1 + y_2 = 15$ —— ①





$\sin 30 = \frac{h_1}{y_1}$

$\Rightarrow \frac{1}{2} = h_1 / y_1$

$\Rightarrow y_1 = 2h_1$ —— ②

Vol. of liquid above initial level = Vol. of fluid added (as vol. below initial line is Constant).

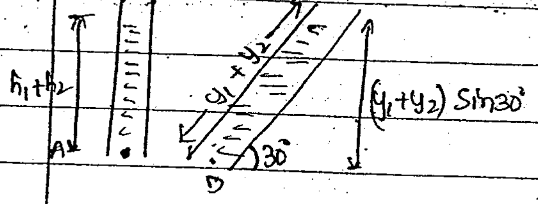
$Ah_2 + Ay_2 = 7.5$

$\Rightarrow A(h_2 + y_2) = 7.5$

$\Rightarrow (h_2 + y_2) = \frac{7.5}{A}$

$\Rightarrow h_2 + y_2 = \frac{7.5}{0.5}$

$\Rightarrow h_2 + y_2 = 15$ —— ③



$\Rightarrow (h_1 + h_2)1.25 = (y_1 + y_2) \sin\alpha$

$\Rightarrow (h_1 + h_2)1.25 = 15 \sin 30°$ from eqn ①

$\Rightarrow h_1 + h_2 = \frac{15 \sin 30°}{1.25} = 6$

$h_1 + h_2 = 6$ ——④

from ① and ③

$y_1 + y_2 = h_2 + y_2$

$\Rightarrow y_1 = h_2$ ——⑤

from eqⁿ ② $y_1 = 2h_1$

from ② and ⑤

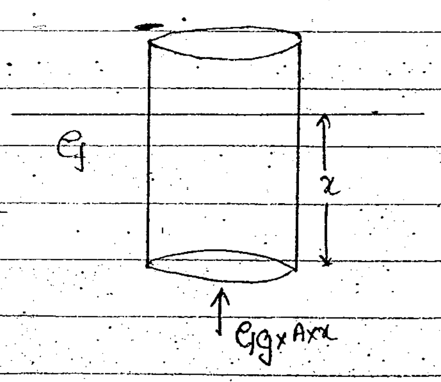
$h_2 = 2h_1 \Rightarrow h_1 = \frac{h_2}{2}$

from ④ $h_1 + h_2 = 6$

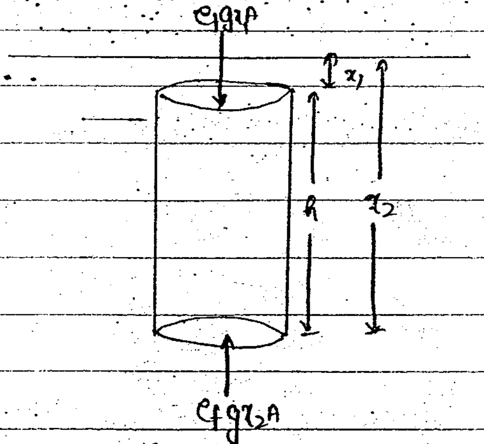
$\Rightarrow \frac{h_2}{2} + h_2 = 6 \Rightarrow h_2 = \underline{4cm}$ Ans.

Buoyancy & Floatation

Partially Submerged



Fully Submerged



$Wt = \omega \times Vol$

$\Rightarrow Wt = \rho g \, vol$

x = depth of immersion

Vol. of body submerged = Ax

$\Rightarrow$ Vol. of body submerged = Vol. of fluid displaced

$= Ax$

$\Rightarrow \rho_f g \times A \times x = f_v$

$\Rightarrow f_v = \rho_f g V_{fd}$

$\Rightarrow f_v = W$

$f_{net} = \rho_f g x_2 A - \rho_f g x_1 A$

$\Rightarrow f_{v\,net} = \rho_f g (x_2 - x_1) A$

$\Rightarrow f_{v\,net} = \rho_f g h A$

as $Ah = V_{fd}$

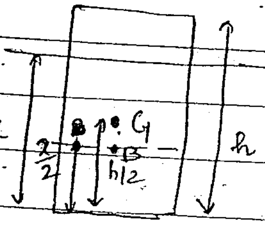
$\Rightarrow \uparrow f_{net} = \rho_f g V_{fd}$

$\uparrow f_{net} = W_{fd} \leftarrow$ Buoyancy force.

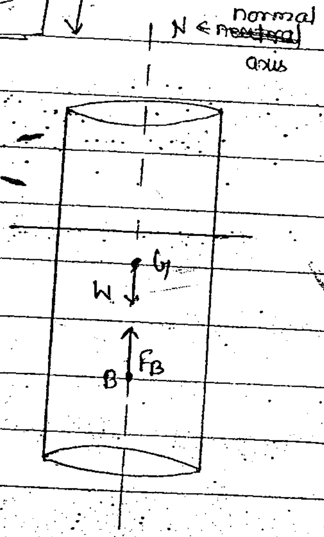
When a body is immersed either partially or completely the net vertical upward force exerted by fluid on the body is known as Buoyancy force and this buoyancy force is equal to weight of fluid displaced and this is known as Archimedes principle. With depth as the pressure increases therefore buoyancy force is basically due to pressure diff.

Centre of Buoyancy (B)

It is the point from which the buoyancy force is suppose to be acting and Centre of buoyancy will lie at centroid of displaced Volume of liquid.



Principle of Floatation



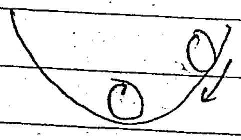
For a floating body in equilibrium, weight of the body is equal to buoyancy force which in turn is equal to weight of fluid displaced and the line of action of these two forces must be same.

$$W_B = F_B$$

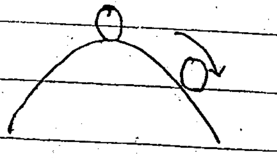
$$F_B = W_{fd}$$

$$\Rightarrow \boxed{W_B = W_{fd}} \; ***$$

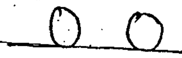
Types of Equilibrium



Stable
Equilibrium



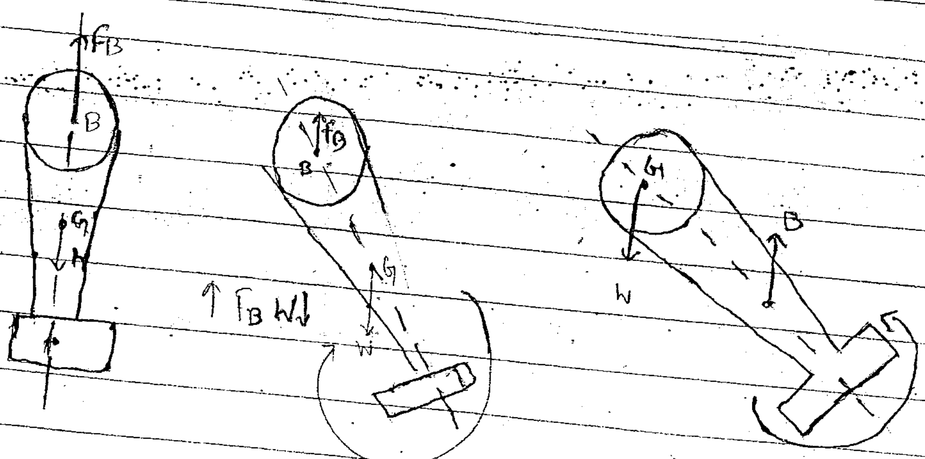
Unstable
Equilibrium



Neutral
Equilibrium

Equilibrium Cond^n^s for Completely Submerged bodies

In tilting Position of G and B donot change as $\rho_f g V_{fd} = F_B$ and ρ_f and V_{fd} remains constant for fully submerged body.



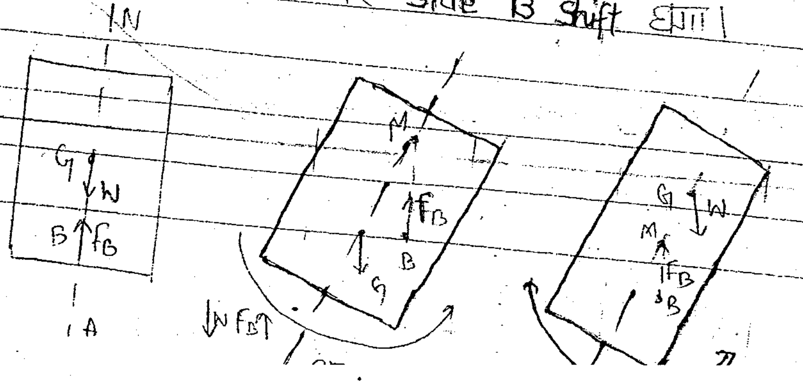
Stable eqlbm

Unstable eqlbm

A Completely Submerged body will be be in stable eqlbm when Centre of buoyancy is above the centre of gravity.
A Completely Submerged body will be in Unstable equilibrium When Centre of buoyancy is below the centre of gravity.
A Completely submerged body will be in neutral eqlbm when G and B Coincides.

Equilibrium Cond^n^s for partially Submerged or floating bodies

Until there is no change in mass Composition there will be no change in Centre of gravity G.
जिधर tilt करेंगे उधर side B shift होगा।



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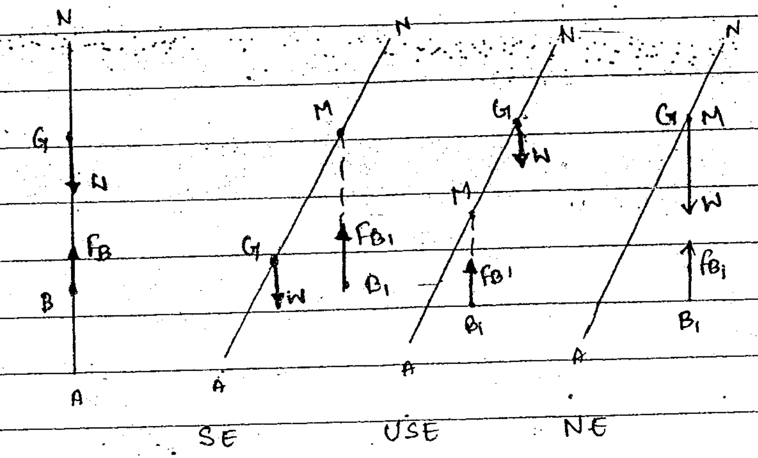
A floating body is in stable equilibrium when meta centre is above the centre of gravity.

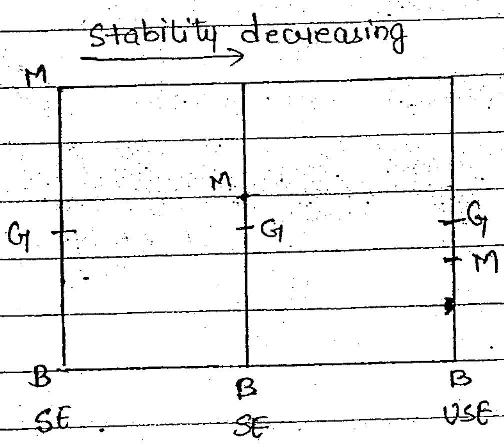
A floating body will be in unstable equilibrium when metacentre is below the centre of gravity.

A floating body will be in neutral equilibrium when 'G' and 'M' coincide.



Metacentric height

The distance b/w centre of gravity (G) and meta centre (M) is known as metacentric height, for stable equilibrium metacentric height is positive, for unstable equilibrium metacentric height is negative and for neutral equilibrium metacentric height is zero.

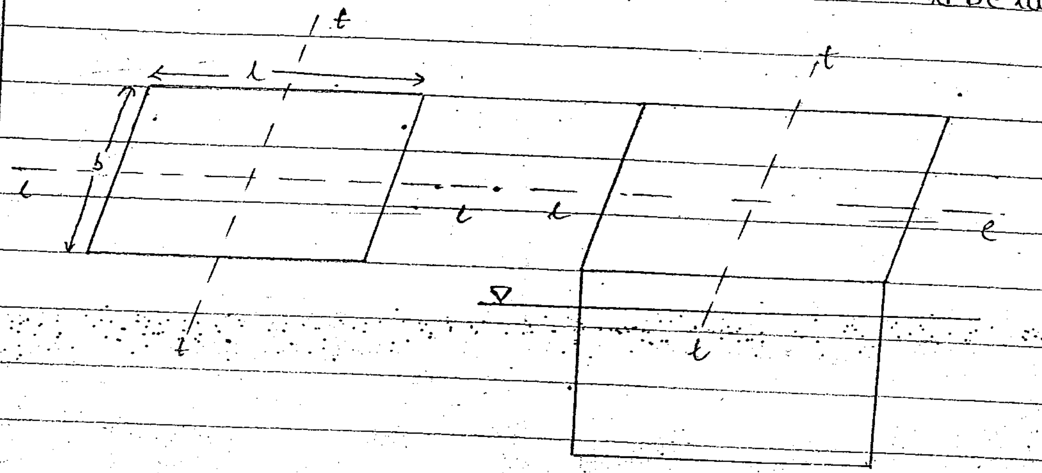


for stable equilibrium BM > BG,

for unstable equilibrium BM < BG,

for neutral equilibrium BM = BG

for more stable equilibrium Conditions (GM) BM should be large.



$I_l = \frac{lb^3}{12}$, $I_t = \frac{bl^3}{12}$

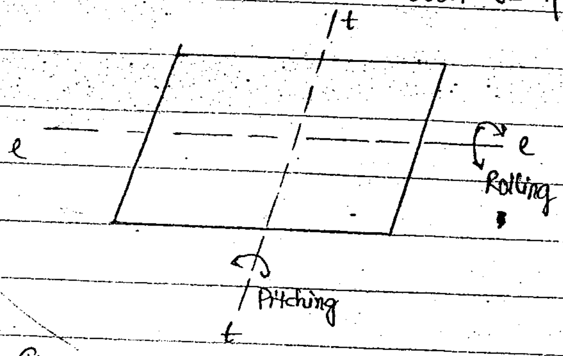
$\Rightarrow I_t > I_l$

$BM = \frac{I}{V}$ = Metacentric height $\Rightarrow BM_t = \frac{I_t}{V}$, $BM_l = \frac{I_l}{V}$

$\Rightarrow BM_t > BM_l$

from design point of view least moment of Inertia is taken into Consideration i.e. about longitudinal axis is taken into Consideration. I is moment of Inertia of the top view area at the free surface.

Oscillation about longitudinal axis is known as rolling, Oscillation about transverse axis is known as pitching.



If Rolling is taken into account then automatically Pitching will be accounted as $BM_t > BM_l$

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Time period for oscillation

$$T = 2\pi\sqrt{\frac{k_g^2}{g(GM)}}$$

k_g = Radius of Gyration
$I = Ak_g^2$
GM = metacentric height

28/12/2011 Though larger GM results in more stable equilibrium Cond^ns but larger GM results in smaller time period of oscillations i.e. frequent oscillations and hence passengers are not comfortable under such Cond^ns leading to sea sickness.

Metacentric height for merchant ships → 0.3m – 1m
Metacentric height for war ships : 1m – 1.5m

Weight Loss in a Liquid due to Buoyancy

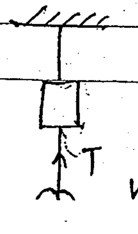
$W_{fd} = \rho_f g V_{fd}$

and $f_B = W_{fd}$

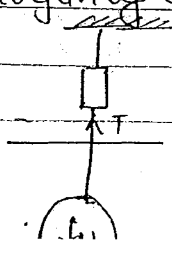
$\Rightarrow f_B = \rho_f g V_{fd}$

air: $1.2\,kg/m^3$
H_2O: $= 10^3\,kg/m^3$

↓ Due to low density buoyancy effects are negligible.



$W = T$



$W = T_1 + f_B$
$\Rightarrow T_1 = W - f_B$

$\text{Wt. loss} = T - T_1$

$\Rightarrow \text{Wt. loss} = W - (W - F_B)$

$= F_B$

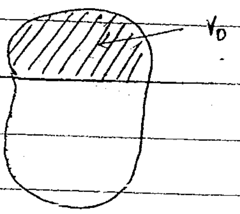
$\Rightarrow \text{Wt. loss} = F_B$

Q32. A body of specific weight 8976 N/m³ extends above the surface of sea water of ~~density~~ specific weight 10104 N/m³ then find the percentage of total volume of the body visible to an observer.

Ans. V = Vol. of body

V_0 = Vol. visible to an observer i.e. above the surface

Vol. submerged $= V - V_0 = V_{fd}$



$\Rightarrow W_B = W_{fd}$

$\Rightarrow \rho_B g V_B = \rho_f g V_{fd}$

$\Rightarrow w_B V_B = w_f V_{fd}$

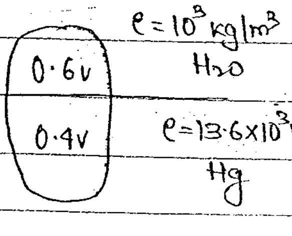
$\Rightarrow 8976\ V = 10104\ (V - V_0)$

$\Rightarrow 8976V = 10104V - 10104V_0$

$\Rightarrow \frac{V_0}{V} = 11.16\%$

Q33. A metallic body flows at the interface of mercury and water in such a way that 40% of its volume is submerged in mercury and 60% in water then find the density of the body.

Ans Let V be the vol. of body



$\Rightarrow W_B = W_{fd}$

$\Rightarrow \cancel{\rho g}\ \rho_B g \times V_B = W_{fd\,Hg} + W_{fd,H_2O}$

$\Rightarrow \rho_B g \times V_B = \rho_{Hg} \times g \times 0.4V + \rho_w \times g \times 0.6V$

$\Rightarrow \rho_B \times V = 13.6 \times 10^3 \times 0.4V + 10^3 \times 0.6V$

$\rho_B = 6040 kg/m^3$

**

Q.34 ~~Q.44~~ A body weighs 100N in air & 80N in water then find the density of the body.

Ans. $T = 100N$

$\Rightarrow T - F_B = 80N$

$\Rightarrow T \cancel{+} - \rho_f \times V \times g = 80$

$\Rightarrow T = 80 + 10^3 \times V \times g$

$\Rightarrow \rho_B \times V \times g = 80 + 10^3 \times V \times g$

~~$\Rightarrow (\rho_B - 1000) \times V \times g =$~~

$F_B = 20N$

$\Rightarrow \rho_f \times V \times g = 20$

$\Rightarrow 1000 \times V \times g = 20$

$\Rightarrow V \times g = \frac{20}{1000}$

$\Rightarrow \rho_B \times V \times g = \frac{\cancel{20}}{\cancel{1000}}\ 100$

$\Rightarrow \rho_B = \frac{\cancel{100}^{5} \times 1000}{\cancel{20}}$ $\Rightarrow \rho_B = 5000 kg/m^3$ Ans.

**

Q35. A body weighs 30N in a liquid of density 800kg/m³ and 15N in a liquid of density 1200 kg/m³ find the volume of the body.

Ans. Let the actual weight of the body be W

$\Rightarrow$ Weight loss by the body $= W - 30$ in liquid $\rho = 800$
$= F_B$ on the body

$\Rightarrow W - 30 = 800 \times g \times (V_B = V_{fd})$ —— ①

Weight loss by the body $= W - 15$ in liquid $\rho = 1200$
$= F_B$ on the body

$\Rightarrow W - 15 = 1200 \times g \times V_B$ ——— (II)

~~(I)~~ = .#

$W = 30 + 800 \times g \times V_B$ ——— (III)

$W = 15 + 1200 \times g \times V_B$ ——— (IV)

(III) = (IV)

$\Rightarrow 30 + 800 \times g \times V_B = 15 + 1200 \times g \times V_B$

$\Rightarrow 15 = 400 \times g \times V_B \Rightarrow V_B = \dfrac{15}{400 \times 9.81} = 3.82 \times 10^{-3} m^3$

Q36.

List - I	List II
A. Stable equilibrium of a floating body	1. Centre of buoyancy below centre of Gravity.
B. Stable equilibrium of a Submerged body	2. 'M' above 'G'
C. Unstable eqlbm of a floating body	3. 'B' above 'G'
D. Unstable eqlbm of a Submerged body	4. 'M' below 'G'

A-2, B-3, C-4, D-1

***

Q37 A metallic Sphere of 0.1 m^3 of Volume and density 2000 kg/m^3 is Completely Submerged in Water and is attached by a wire to a float of Volume 1 m^3 and its density is 400 kg/m^3 calculate the tension in the Wire and Volume of the float i.e. Submerged in Water.

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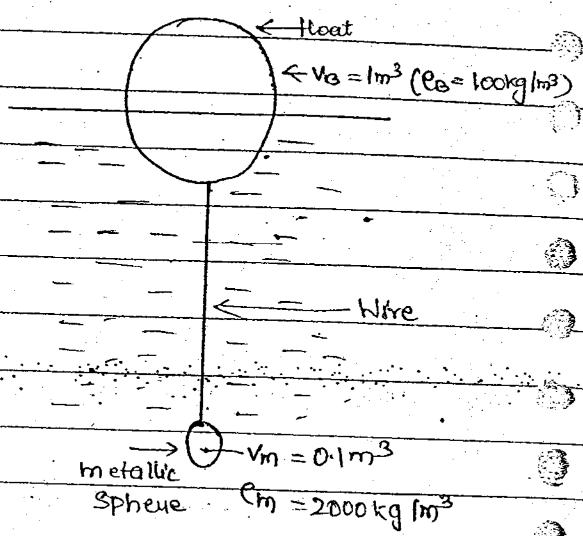
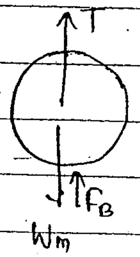
Ans. Metallic Sphere



$W_m = T + F_B$

$\Rightarrow T = W_m - F_B$

$T = \rho_m g V_m - \rho_f g V_{fd}$



$\Rightarrow T = g V_m \rho_m - \rho_f g V_m$

$= g V_m (\rho_m - \rho_f)$

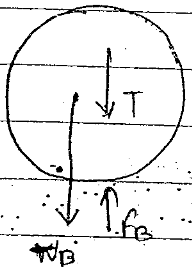
$\Rightarrow T = 9.81 \times 0.1 (2000 - 1000)$

$\Rightarrow T = 981 N$

Float

$W_B + T = F_B$

$\rho_B g V_B + 981 = \rho_f g V_{fd}$



$\Rightarrow 100 \times 9.81 \times 1 + 981$

$= 10^3 \times 9.81 \times V_{fd}$

$\Rightarrow V_{fd} = 0.2m^3$ (i.e. vol. of float submerged)

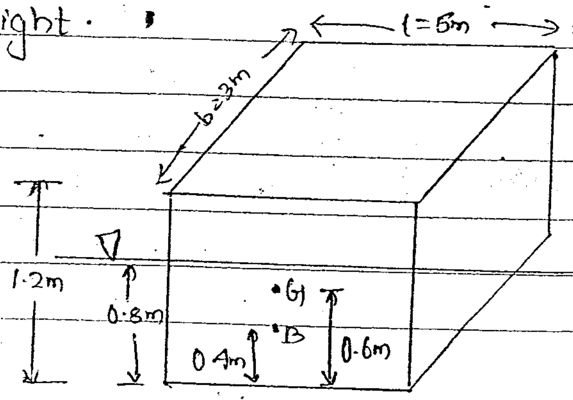
Q38 A rectangular body 5m long 3m wide and 1.2m height is immersed to a depth of 0.8m in water the density of water is 1025 kg/m³ Then find metacentric height.

Ans. $BM = \frac{I}{V}$

V = vol. of fluid displaced

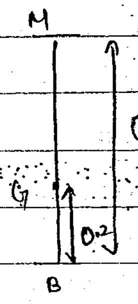
$V = 5 \times 0.8 \times 3$

$I_{least} = \frac{5 \times 3^3}{12}$



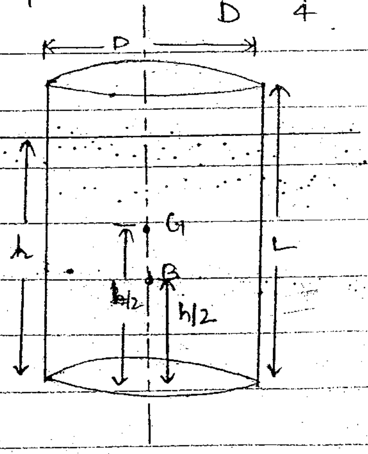
$BM = \frac{5\times3^3}{12} \times \frac{1}{5\times3\times0.8}$

$\Rightarrow BM = 0.9375\,m$



$\Rightarrow MG = 0.7375\,m$ Ans

Q.39 A cylinder of density 600kgm/m³ floats in oil of density 900kg/m³ with its longitudinal axis vertical. If L = height & D = diameter of the cylinder then show that for stable equilibrium $\frac{L}{D} < \frac{3}{4}$



Ans: $W_B = W_{fd}$

$\rho_B \times g \times V_B = \rho_f \times g \times V_{fd}$

$\rho_B \times \frac{\pi}{\cancel{4}} \times \cancel{D^2} \times L = \rho_f \times g \times \frac{\pi}{\cancel{4}} \cancel{D^2} \times h$

$\Rightarrow 600 \times L = 900 \times h$

$\Rightarrow h = \frac{2L}{3}$

$BG = \frac{L}{2} - \frac{h}{2} \Rightarrow BG = \frac{L}{2} - \frac{L}{3} = \frac{L}{6}$

$\Rightarrow BG = \frac{L}{6}$

$BM = \frac{I}{V} \leftarrow$ vol. of fluid displaced $\Rightarrow I = \frac{\pi}{64} D^4$

$\Rightarrow V = V_{fd} = \frac{\pi}{4} D^2 \times h = \boxed{\frac{\pi}{4} \times D^2 \times \frac{2L}{3}}$

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$$V = \frac{\pi D^2 L}{6}$$

$$BM = \frac{\pi D^4 \times 6}{64 \times \pi D^2 L} = \frac{3D^2}{32L}$$

for Stability

$BM > BG$

$$\Rightarrow \frac{3D^2}{32L} > \frac{L}{6}$$

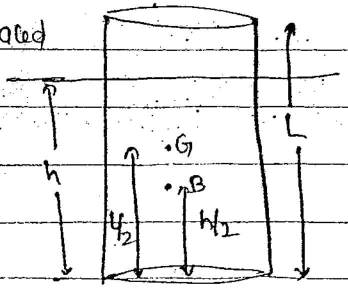
$$\Rightarrow \frac{9}{16} > \frac{L^2}{D^2}$$

$$\Rightarrow \frac{3}{4} > \frac{L}{D}$$

$$\Rightarrow \frac{L}{D} < \frac{3}{4}$$

Q40. A solid cylinder of length 'L' and diameter 'D' and density $600 kg/m^3$ floats in neutral equilibrium in water with its axis vertical then find $\frac{L}{D}$;

Ans: Weight of body = Weight of liquid displaced



$$\Rightarrow 600 \times \frac{\cancel{\pi}}{\cancel{4}} \times \cancel{D^2} \times L = 1000 \times \frac{\cancel{\pi}}{\cancel{4}} \times \cancel{D^2} \times h$$

$$\Rightarrow \frac{L}{h} = \frac{10}{6} = \frac{5}{3}$$

$$\Rightarrow h = \frac{3L}{5}$$

$$\Rightarrow B \cdot G = \frac{L}{2} - \frac{h}{2} = \frac{L}{2} - \frac{3L}{10} = \frac{2L}{10} = \frac{L}{5}$$

$$B.M = \frac{\frac{\pi}{64} D^4}{\frac{\pi}{4}\times D^2 \times h} = \frac{\cancel{\pi}\times\cancel{4}\, D^{\cancel{4}\,2}}{\underset{16}{\cancel{64}}\times\cancel{\pi}\,\cancel{D^2}\times h}$$

$$= \frac{D^2}{16h} = \frac{D^2\times 5}{16\times 3L} = \frac{5D^2}{48L}$$

for neutral eqlb^m.

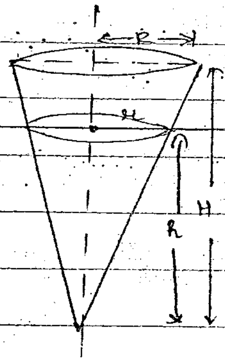
$$B.M. = G.B$$

$$\Rightarrow \frac{L}{5} = \frac{5D^2}{48L}$$

$$\Rightarrow \frac{L^2}{D^2} = \frac{25}{48}$$

$$\Rightarrow \frac{L}{D} = \underline{\underline{0.721}} \text{ Ans.}$$

Q41. A Cone floats in water with its apex downwards as shown in fig. then find the Cond^n for Stable equilibrium.



Ans. By Similar triangles

$$\frac{r}{h} = \frac{R}{H}$$

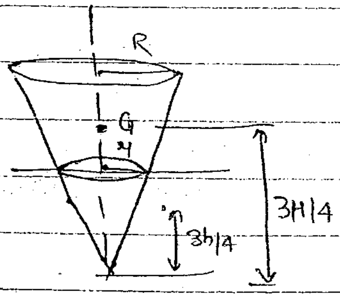
$$\Rightarrow r = \frac{R\times h}{H}$$

$$W_B = W_{fd}$$

$$\rho_B \cancel{g} V_B = \rho_f \cancel{g} V_{fd}$$

$$\Rightarrow \rho_B \times \frac{1}{\cancel{3}}\cancel{\pi} R^2 H = \rho_f \times \frac{1}{\cancel{3}}\cancel{\pi} r^2 h$$

$$\Rightarrow R^2 H = \frac{\rho_f}{\rho_B} r^2 h$$



$$BG = \frac{3H}{4} - \frac{3h}{4}.$$

$$BM = \frac{I}{V} \quad \text{Where } I = \frac{\pi d^4}{64} = \frac{\pi}{64}(2r)^4$$

$$V = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad BM = \frac{\pi}{64}\frac{(2H)^4}{\pi H^2 \times h} \times 3 = \frac{3\cancel{\pi} r^{4\,2}}{4\cancel{\pi}\cancel{r^2}h} = \frac{3r^2}{4h}$$

$$r = \frac{R}{H} \times h$$

$$\Rightarrow \quad 3 \times \frac{R^2}{H^2} \times \frac{h^{\cancel{2}}}{4\cancel{h}} = \frac{3R^2}{4H^2} \times h$$

$$\therefore \quad R^2 H = \frac{e_f}{e_B} \times r^2 \times h$$

$$\Rightarrow \quad \cancel{R^2} H = \frac{e_f}{e_B} \times \frac{\cancel{R^2}}{H^2} \times h^3$$

$$\Rightarrow \quad h = \left(\frac{e_B}{e_f}\right)^{1/3} \times H$$

$$\Rightarrow \quad BG = \frac{3H}{4} - \frac{3h}{4}$$

$$= \cancel{\frac{3}{4}\left(\left(\frac{e_B}{e_f}\right)\right.} \quad \frac{3}{4}\left(1 - \left(\frac{e_B}{e_f}\right)^{1/3}\right) H$$

for stable eqlbm

$BM > BG$

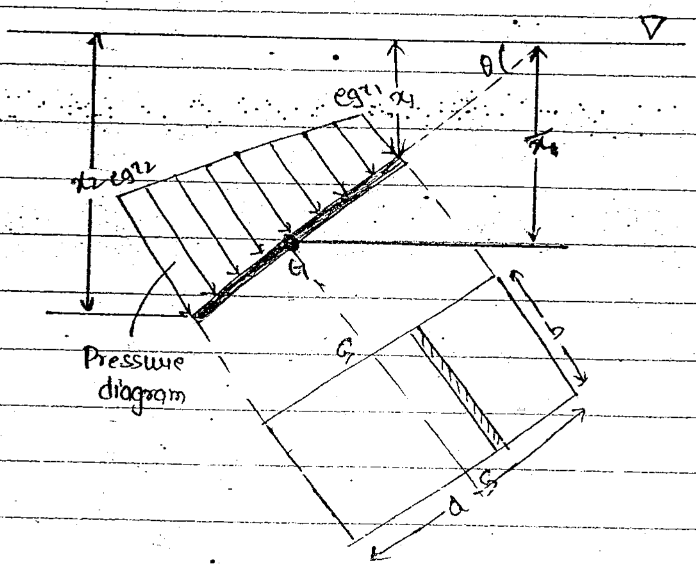
$$\Rightarrow \quad \frac{\cancel{3}}{\cancel{4}} \times \frac{R^2}{H^2} \times h > \frac{\cancel{3}}{\cancel{4}}\left(1 - \left(\frac{e_B}{e_f}\right)^{1/3}\right) H$$

$$\Rightarrow \quad \boxed{h > \frac{H^3}{R^2}\left(1 - \left(\frac{e_B}{e_f}\right)^{1/3}\right)}$$

Hydrostatic Forces

Hydrostatic forces on plane Surfaces

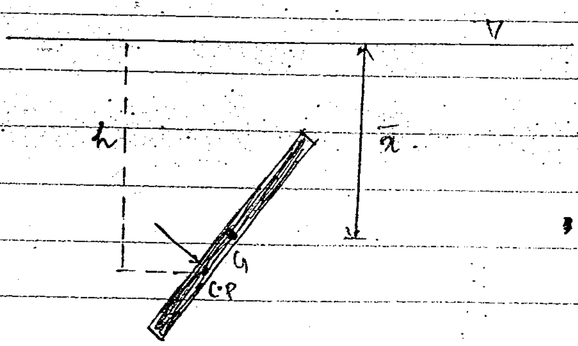
Case-I Plane inclined surface



hydrostatic force $F = wA\bar{x}$

The point from which the hydrostatic force is suppose to be acting is known as Centre of pressure.

From principle of moments,



$$\boxed{h = \bar{x} + \frac{I_G}{A\bar{x}} \sin^2\theta}$$

I_G - Moment of Inertia about Centroidal axis (G, G.)

The Centre of pressure is below the Centre of Gravity because pressure increases with depth

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Case-II Plane vertical Surface

Put $\theta = 90^\circ$ in previous Case

$$F = wA\bar{x}$$

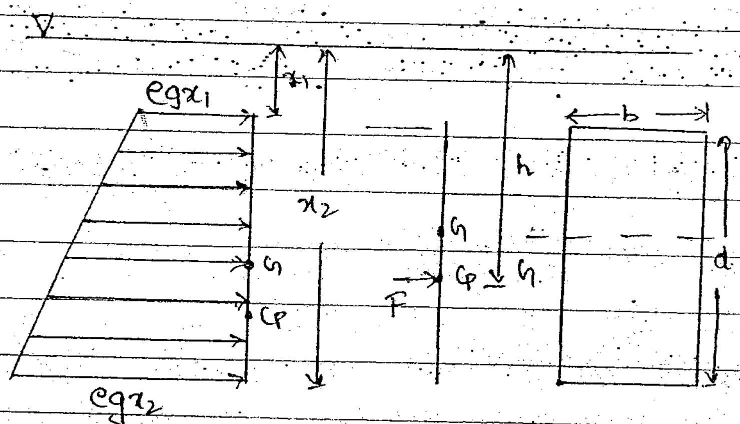
$$h = \bar{x} + \frac{I_G}{A\bar{x}}\sin^2\theta$$

$$\Rightarrow F = wA\bar{x}$$

$$\& \; h = \bar{x} + \frac{I_G}{A\bar{x}}\sin^2 90^\circ$$

$$\Rightarrow h = \bar{x} + \frac{I_G}{A\bar{x}}$$

I_G is moment of Inertia about Centroidal axis which is parallel to free Surface.



$$I_G = \frac{bd^3}{12}$$

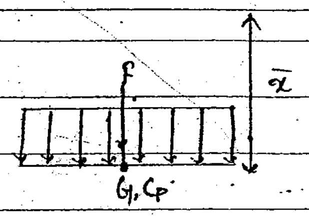
** In Case of Vertical Surfaces, Centre of pressure and Centre of gravity are almost same when depth of immersion $\bar{x}$ is large.

Case-III Plane Horizontal Surface

Put $\theta = 0^\circ$ in Case (i)

$f = wA\bar{x} \longrightarrow f = wA\bar{x}$

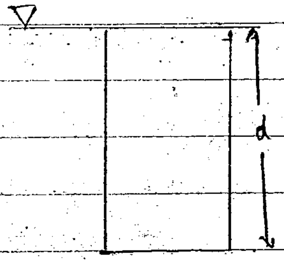
$h = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}} \longrightarrow h = \bar{x}$



Case	Force, F	CP
Inclined	$wA\bar{x}$	$h = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}$
Horizontal	$wA\bar{x}$	$h = \bar{x}$
Vertical	$wA\bar{x}$	$h = \bar{x} + \frac{I_G}{A\bar{x}}$

Q42. List I (Surface) List II (C.P.)

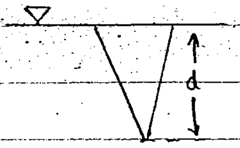
A).



1. $\frac{5d}{8}$

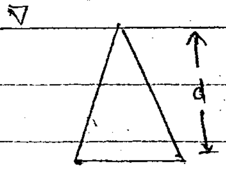
2. $\frac{3d}{4}$

B).



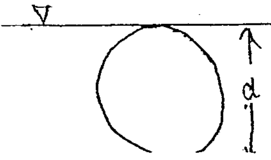
3. d/2

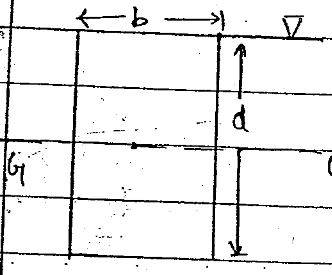
c).



4. 2d/3

D).





$\bar{x} = d/2$

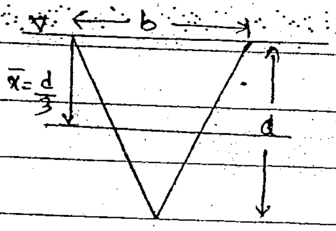
$I_G = \frac{bd^3}{12}$

$A = bd$

$\Rightarrow h = \bar{x} + \frac{I_G}{A\bar{x}}$

$= \frac{d}{2} + \frac{bd^3}{12} \times \frac{1}{bd} \times \frac{1}{d/2}$

$\Rightarrow \boxed{h = \frac{2d}{3}}$



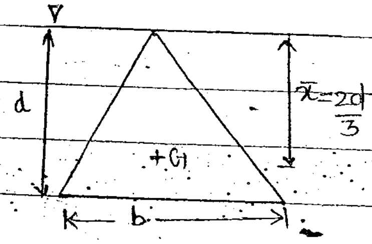
$A = \frac{1}{2}bd$

$I_G = \frac{bd^3}{36}$

$\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}}$

$\Rightarrow h = \frac{d}{3} + \frac{bd^3}{36} \times \frac{1}{\frac{1}{2}bd} \times \frac{1}{d/3}$

$\Rightarrow \boxed{h = \frac{d}{2}}$



$\bar{x} = \frac{2d}{3}$

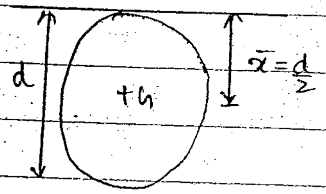
$I_G = \frac{bd^3}{36}$

$A = \frac{1}{2}bd$

$h = \bar{x} + \frac{I_G}{A\bar{x}}$

$= \frac{2d}{3} + \frac{bd^3}{36} \times \frac{1}{\frac{1}{2}bd} \times \frac{1}{2d/3}$

$\boxed{h = \frac{3d}{4}}$



$\bar{x} = d/2 \,;\, A = \frac{\pi}{4}d^2$

$I_G = \frac{\pi}{64}d^4$

$\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}}$

$\Rightarrow \boxed{h = \frac{5d}{8}}$

29/12/2011

Q42. A vertical gate 6m × 6m holds water on one side with free surface at it's top then find the moment about the bottom edge of the gate due to water force. Take w = specific

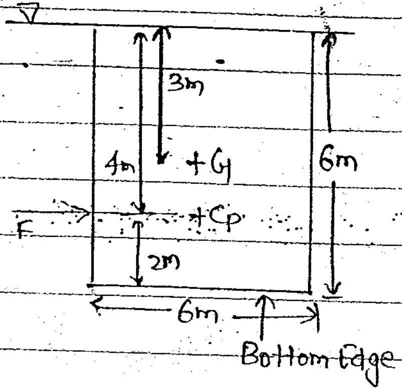
Weight of Water.

(a) 36 w · (b) 72 w (c) 216 w (d) 108 w

Ans. $f = wA\bar{x}$

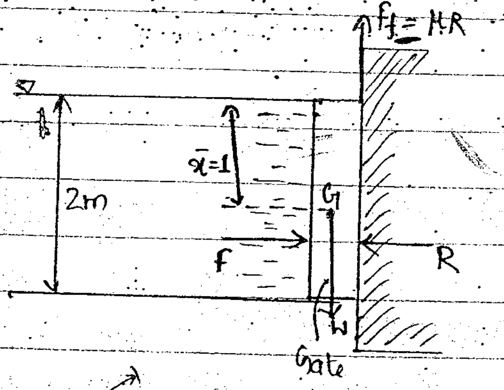
$= w \times 6 \times 6 \times 3$

$= 108\, w$



$\Rightarrow$ Moment $= F \times 2 = 108 \times 2\, w = 216\, w$

Q43. A vertical Gate 2.5m wide and weighing 500kg is held in position due to horizontal force on one side and associated friction as shown in fig. When the water level drops down to 2m above the bottom of the Gate, the Gate just starts sliding down; then find the coefficient of friction b/w Gate and supporting structure.



Ans. Hydrostatic force $= F$

$F = R \rightarrow$ normal rx^n

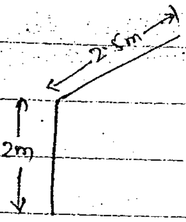
$wt = f_f$

$\Rightarrow wt = \mu R$

$\Rightarrow wt = \mu F$

$wt = mg$

$\Rightarrow wt = 500 \times g$



$F = wA\bar{x}$

$= w \times 2 \times 2.5 \times 1$

$= 5w$

$= 5 \times 10^3 \times g$

$\Rightarrow 500 \times \cancel{g} = \mu \times 5 \times 10^3 \times \cancel{g} \Rightarrow \underline{\underline{\mu = 0.1}}$ Ans

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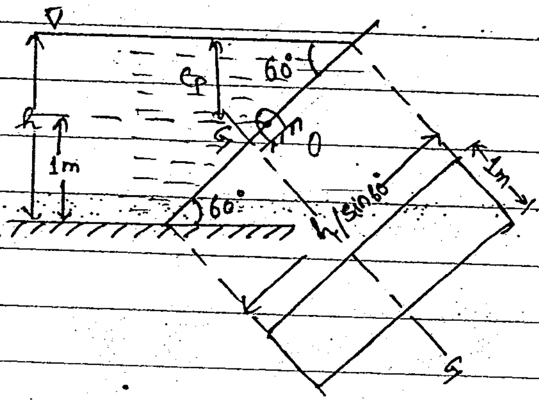
Q44. The automatic tipper as shown in fig. operates when the water level reaches certain height. calculate the level when it is about to tip.

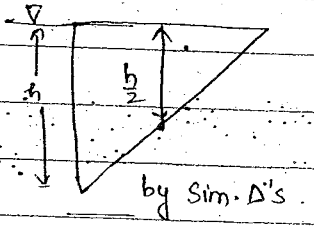
Ans. The tipper will tip when the centre of pressure is above point O, and for critical cond^ns the centre of pressure must be at point O.



$C_p = (h-1)\,m$ —— ①

जहाँ पर hydriastotic force लग रहा है उसका C.G. लेना है बाकी को Consider नहीं करना है।

$\bar{x} = \frac{h}{2}$



$A = 1 \times 1.154h$

$\Rightarrow A = 1.154h$

$I_G = \frac{1 \times (1.154h)^3}{12}$

$C_p = \bar{x} + \frac{I_G}{A\bar{x}} \sin^2\theta$

$\Rightarrow C_p = \frac{h}{2} + \frac{1 \times (1.154h)^3}{12} \times \frac{1}{1.154\,h} \times \frac{1}{h/2} \times \sin^2 60$

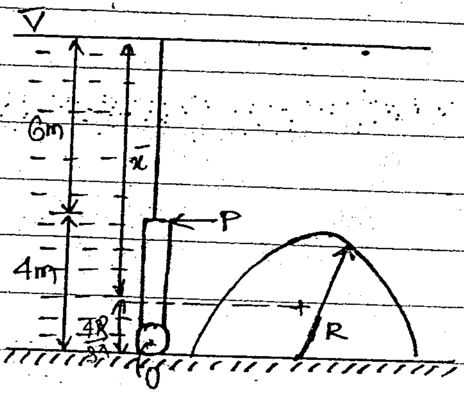
$\Rightarrow C_p = 0.6664\,h$ —— ②

$\Rightarrow C_p = h - 1$ —— ①

from ① and ②

$\Rightarrow h - 1 = 0.6664h \Rightarrow h = 2.99\,m \approx 3\,m$

Q45. A Semi Circular plate hinged at O is held at place by a horizontal force P acting at A as shown in fig. Calculate the force P reqd for equilibrium. For Semicircular plate the distance of Centre of Gravity from straight edge is $\frac{4R}{3\pi}$ and the moment of Inertia about Centroidal axis is $I_G = 0.035\pi R^4$



Ans. $\bar{x} = 10m - \frac{4\times R}{3\pi}$

$= 10m - \frac{4\times 4}{3\times\pi}$

$\Rightarrow \bar{x} = 8.3 m$

$\Rightarrow F = wA\bar{x}$

$= 9810 \times \frac{\pi}{2} \times (4)^2 \times 8.3$

$= \frac{4092.766 kN}{2} = 2046.38 kN.$

$\Rightarrow h = \bar{x} + \frac{I_G}{A\bar{x}} = 8.3 + \frac{0.035\pi\times(4)^4}{(\pi/2)\times R^2\times 8.3}$

$\Rightarrow h = 8.435 m$

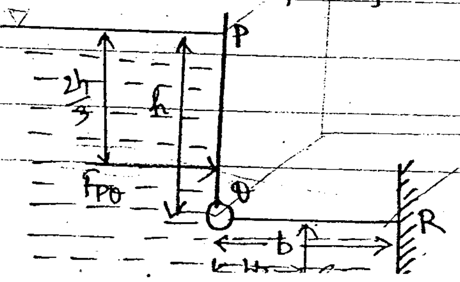
$\Rightarrow$ Taking moment about O,

$\Rightarrow F\times(10-8.435) = P\times 4$

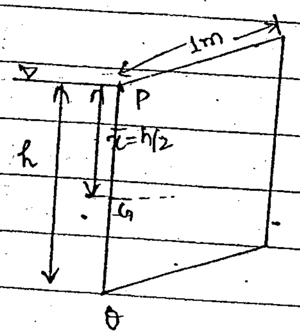
$\Rightarrow 2046.38\times 1.565 = P\times 4$

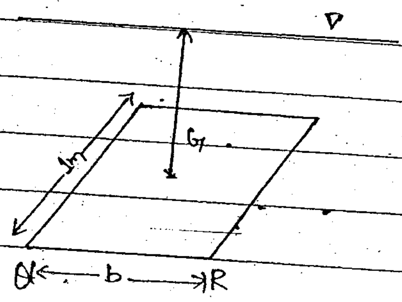
$\Rightarrow P = 800.6 kN$

Q46. The fig. Shows a hydraulic Gate PQR whose weight is very small compared to hydrostatic forces. find the value of 'h' for equilibrium:



Ans





$F_{PO} = wA\bar{x}$

$F_{PO} = w \times h \times 1 \times \frac{h}{2}$

$\Rightarrow F_{PO} = \frac{wh^2}{2}$

$F_{OR} = w \times b \times 1 \times h$

$= wbh$

$F_{PO} \times \frac{h}{3} = F_{OR} \times \frac{b}{2}$

$\Rightarrow \frac{wh^2}{2} \times \frac{h}{3} = wbh \times \frac{b}{2}$

$\Rightarrow \frac{h^2}{3} = b^2 \Rightarrow h = \sqrt{3}b$

Hydrostatic Forces on Curved Surfaces

Flat Surfaces में dirn of force throughout is constant

$dF = P \times dA$

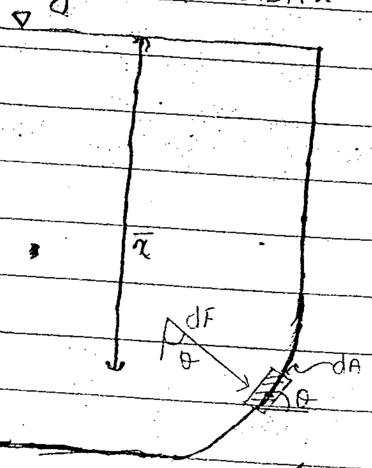
$P = \rho g x \Rightarrow dF = \rho g x\, dA$

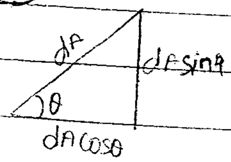
$dF_H = dF\sin\theta$

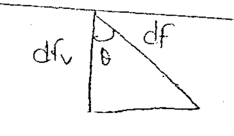
$dF_V = dF\cos\theta$

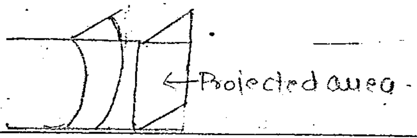
$\Rightarrow dF_H = dF\sin\theta$

$= \rho g x\, \underbrace{dA\sin\theta}_{\text{vertical projection area}}$









Horizontal Component of force on Curved Surface is equal to hydrostatic force on vertical projection area and this force will act at the Centre of pressure of Corresponding area.

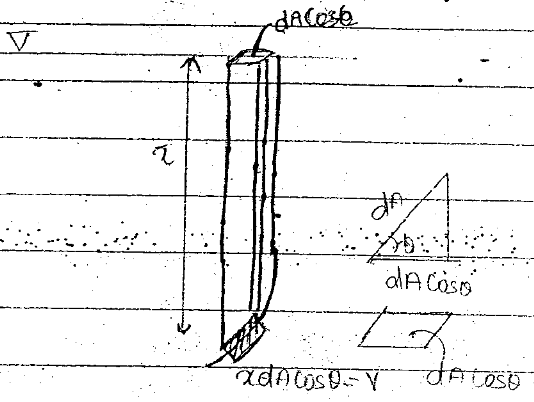
$dF_v = dF\cos\theta$

$dF = \rho g x\, dA$

$dF_v = \rho g x\, dA\cos\theta$

$dF_v = \rho g V$

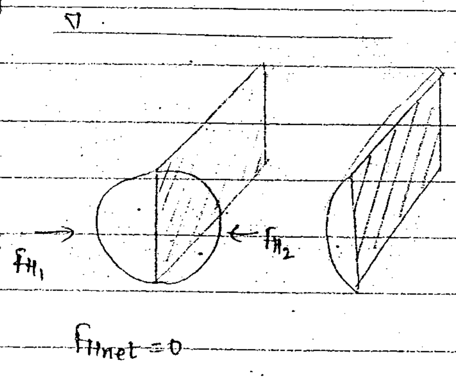
$dF_v = $ wt. of fluid



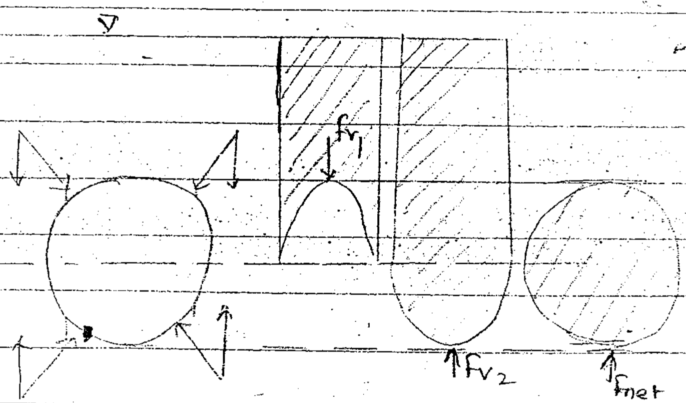
Vertical Component of force on Curved Surface is equal to Weight of the fluid Contained by the Curved surface taken upto its free Surface and this force will act at the C.G. of Corresponding Weight.

Special Case - I

When a Cylinder is Completely immersed in water.

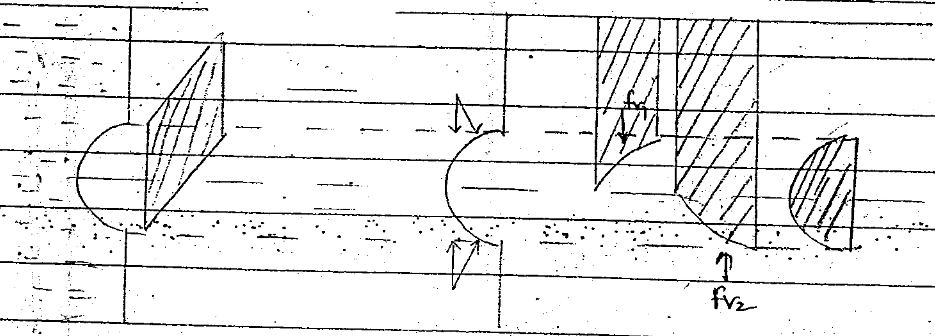


$F_{Hnet} = 0$



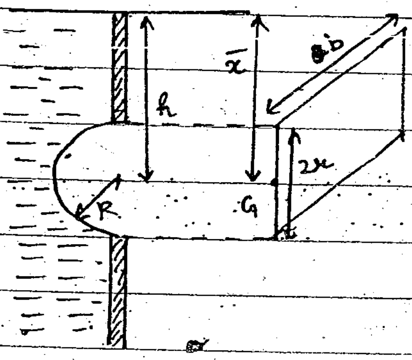
$F_{net} = F_{V_2} - F_{V_1} = $ wt. of fluid Corresponding to Cylindrical vol.

Case-II Semicircular Gate Subjected to Water force



Q47. find the horizontal and vertical Components of force on Semi-Circular gate having width 'b'

Ans.



$\bar{x} = h$

$F_H = wA\bar{x}$

$F_H = w \times b \times 2r \times h$

$\Rightarrow F_H = 2wbrh$

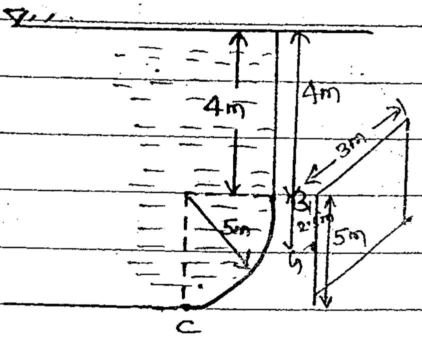
$F_v =$ Wt. of fluid displaced

$F_v = \rho g V$

$\Rightarrow F_v = \rho g \frac{\pi r^2 b}{2}$ [Bouyant force]

Q48. The tank in the fig is 3m wide, calculated the hydrostatic, horizontal, vertical and resultant force on 1/4th of a Circle BC.

Ans.



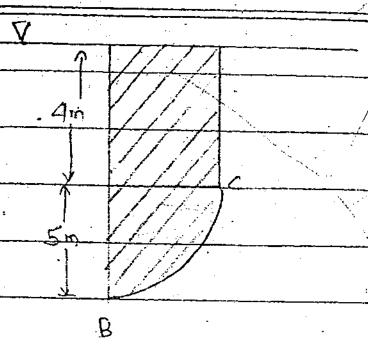
$F_H = wA\bar{x}$

$F_H = 9810 \times 5 \times 3 \times 6.5$

$\Rightarrow F_H = 956.4 \times 10^3 N$

$F_H = 956.47KN$

$F_v = wt = \rho g v$



$f_v = \rho g v \Rightarrow f_v = 10^3\times 9.81\left[4\times5\times3 + \frac{\pi(5)^2\times3}{4}\right]$

$V = 4\times5\times3 + \frac{\pi 5^2}{4}\times3$

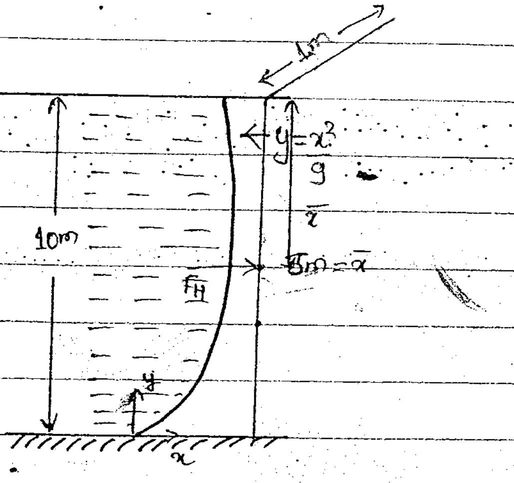
$\Rightarrow f_v = 1166.45 kN$

Q49. Find the magnitude and dirxⁿ of water force acting on curved surface of dam which is shaped according to eqⁿ $y = \frac{x^2}{9}$ as shown in fig. The height of water retained by dam is 10m. The width of the dam is 1m.

Ans. $f_H = wA\bar{x}$

$= 10^3\times1\times10\times5\times9.81$

$= 490.5\times10^3 N$



Area of water under the curve

$\Rightarrow \cancel{y} \; x = 3\sqrt{y}$

$\Rightarrow dA = x.dy = 3\sqrt{y}.dy$

$\Rightarrow A = \int_0^{10} 3\times\frac{2}{\cancel{3}}\times \cancel{y^{3/}} \; 3\sqrt{y}.dy$

$= \cancel{3}\times\frac{2}{\cancel{3}}\times y^{3/2}\Big|_0^{10} = 2\times y^{3/2}\Big|_0^{10}$

$= 63.24 m^2$

$\Rightarrow$ Wt. of liquid acting on curved surface $= 10^3\times9.81\times 63.24\times1$

$\Rightarrow f_v = 620384.4$

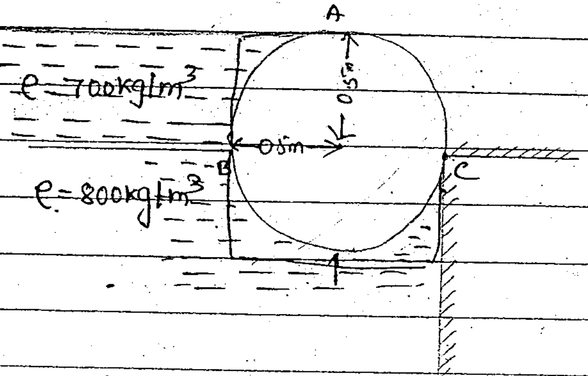
$\Rightarrow F = \sqrt{F_H^2+F_v^2} = 790.864$ KN Ans.

$\tan\theta = \frac{F_V}{F_H} = \frac{620.3}{490.5} \Rightarrow \theta = 51.6°$

****

Q50. A cylinder of 1m diameter and length 2m stays in equilibrium as shown in fig. Calculate the density of cylinder

Ans. Wt cylinder = ρg vol.

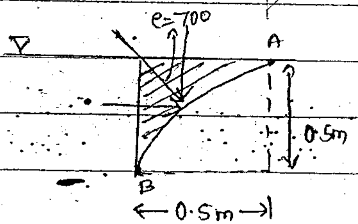


Wt. cylinder $= \rho_{cyl} \times 9.81 \times \frac{\pi}{4}(1)^2 \times 2$

$\Rightarrow$ Wt. cylinder $= 15.41\, \rho_{cyl} \downarrow$

$F_{V_{A,B}} =$ Wt $= \rho g$ vol.

Vol $= \left[0.5 \times 0.5 - \frac{1}{4} \times \pi (0.5)^2\right] \times 2$



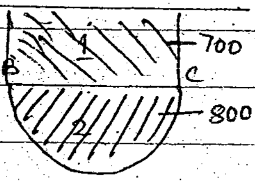
$F_{V_{A,B}} = 700 \times 9.81 \left[0.5 \times 0.5 - \frac{1}{4}\pi(0.5)^2\right] \times 2$

$F_{V_{A,B}} =$ ~~763.8 N~~ $\downarrow$ 736.8 N $\downarrow$

$700 \times h_1 = 800 \times h_2$

$\boxed{h_2 = \frac{7}{8} h_1}$

$F_1 = 700 \times 9.81 \times 1 \times 0.5 \times 2$



$F_2 = 800 \times 9.81 \times \frac{\pi}{4}(1)^2 \times \frac{1}{2} \times 2$

$F_{V_{A,B,C}} = 13030.8$

Considering only portion BC.

$800 \times 9.81 \times \left[0.5 \times 0.5 \times \frac{7}{8}\right] + 1 \times 2$ (Area projection of base) $\Rightarrow$

$\Rightarrow$ Wt + $F_{V_{AB}} = F_{V_{BC}}$

$- 800 \times 9.81 \times \left[0.5 \times 1 - \frac{\pi \times 0.5^2}{2} \times 2\right] \Rightarrow$

$= 13030.8$

$15.41\, \rho_{cyl} + 736.8 = 13030.8$

$\rho_{cyl} = 797.7$

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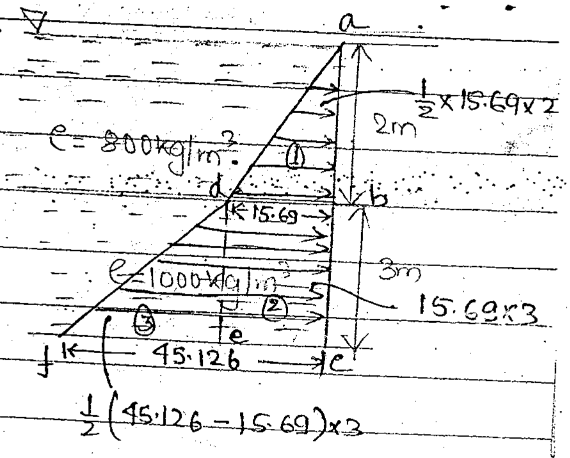
Q51. A tank Contains Water of density 1000kg/m³ ~~upto~~ a height of 3m above the base and immiscible liquid of density 800kg/m³ is filled on top of that over two metre depth. Calculate the force on vertical wall of width 6m.



Ans. $P_c = 800 \times 9.81 \times 2 + 1000 \times 9.81 \times 3$

$P_c = 45.126\ kN/m^2$

$P_B = 800 \times 9.81 \times 2$

$\Rightarrow P_B = 15.69 \times 10^3 N/m^2$

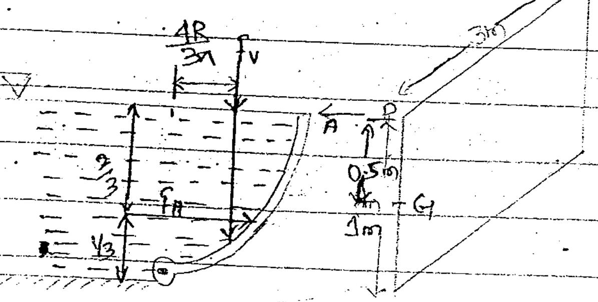
$= 15.69\ kN/m^2$

$\text{force/width} = \frac{1}{2} \times 15.69 \times 2 + 15.69 \times 3 + \frac{1}{2}(45.126 - 15.69 \times 3) \times 3$

$= 106.94\ kN/m$

$\Rightarrow \text{force} = 106.94 \times 6 = 641.4\ KN$

Q52. The Gate OA as shown in fig is hinged at O ~~and~~ is in the form of 1/4th of a circle of 1m radius it supports water on one side, the width of the Gate is 3m. Find the force P reqd. to hold the gate in position.



Ans. 4 Golden points

- F_H is calculated by projecting area on vertical plane.
- Vertical component is weight of liquid above ~~free~~ curved surface.
- F_H acts on the Cp of projected area.
- F_V acts on the Cg of the body

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$f_H = wA\bar{x}$

$= 9810 \times 1 \times 3 \times 0.5$

$= 14.7 KN$

$f_V = \dot{c}\dot{o}t = \rho g V$

$= 10^3 \times 9.81 \times \frac{\pi (1)^2}{4} \times 3$

$\Rightarrow f_V = 23.11 kN.$

Taking moment about O.

$f_H \times \frac{1}{3} + f_V \times \frac{4 \times 1}{3\pi} = P \times 1$

$\Rightarrow 14.7 \times \frac{1}{3} + 23.11 \times \frac{4}{3\pi} = P$

$\Rightarrow P = 14.7 KN$ Ans.

Fluid Kinematics.

Kinematics deals with motion of fluid without any reference to forces

fluid flow analysis is done with the help of

- Langrangian Approach
- Eulerian Approach

In Langrangian approach a single fluid particle is taken and behaviour of this fluid particle is analysed.

In Eulerian technique certain section is taken in space and behaviour of various fluid particles is analysed at this section. As it is easy to analyse at a section Eulerian technique is mostly used in fluid flow analysis

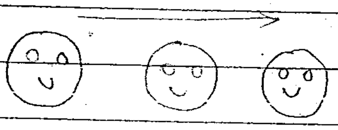
Types of Fluid Flow

- Steady and Unsteady flow
  A flow is said to be a steady flow when fluid properties do not vary w.r.t time at any given section, otherwise the flow is unsteady. (for steady flow $\frac{dv}{dt} = 0$, $\frac{dP}{dt} = 0$ etc.)
- Uniform and Non-Uniform flow (Time के साथ नहीं space के साथ बदलती)
  A flow is said to be uniform flow if the velocity remains constant at different sections at any given instant of time, otherwise the flow is non-uniform. (For Uniform flow $\frac{dv}{ds} = 0$)
- Laminar and Turbulent flow
  In laminar flow fluid particles move in the form of layers with one layer sliding over other. Laminar flow generally occurs at

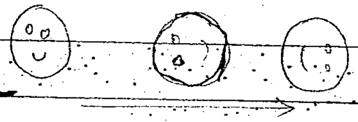
low velocities. When fluid particles move in highly disorganised manner leading to rapid mixing of particles that flow is known as turbulent flow. Turbulent flow generally occurs at high velocities.

Rotational and Irrotational flow

A flow is said to be rotational flow when fluid particles rotate about their own mass centres, otherwise the flow is irrotational. In case of irrotational flow, there is no rotation and hence there is no torque i.e. there is no tangential force (shear force) and this occurs generally in non-viscous fluids.



Irrotational



Rotational

Stream Line

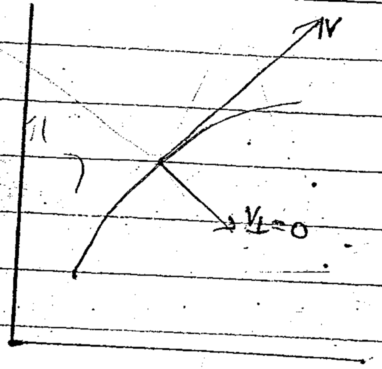
It is an imaginary line or curve drawn in space such that a tangent drawn to it at any point gives velocity vector. (i.e. dir^n of velocity)

यहाँ पर Photo खींचे।

Stream line is an instantaneous snapshot of flow. There is always flow along the stream line and there is no flow across the stream line (as ⊥ comp. of velocity is zero)

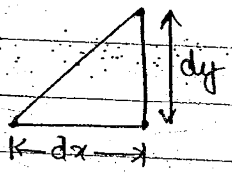
No two stream lines can intersect or a single stream line intersect with each other because at any point velocity must be unique at a point

Equation of a stream line



$\vec{V} = U\hat{i} + V\hat{i} + \omega\hat{k}$

$\Rightarrow |\vec{V}| = \sqrt{u^2 + v^2 + \omega^2}$



$U = \frac{dx}{dt}$, $v = \frac{dy}{dt}$

$\Rightarrow dt = \frac{dx}{u}$, $dt = \frac{dy}{v}$

$\Rightarrow \boxed{\frac{dx}{u} = \frac{dy}{v}}$ → eqn of a stream line in 2-D

→ $vdx - udy = 0$

$\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{\omega}}$ → eqn of stream line in 3-D.

Stream line flow means organized flow.

Q53. A flow is represented by $\vec{V} = ax\hat{i} + ay\hat{j}$ where a is constant then find the eqn of a stream line passing through (1,2)

Ans. $\vec{V} = ax\hat{i} + ay\hat{j}$

Comparing with $u = ax$, $v = ay$

$\Rightarrow \frac{dx}{u} = \frac{dy}{v}$

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$\Rightarrow \frac{dx}{\cancel{a}x} = \frac{dy}{\cancel{a}y}$

$\Rightarrow \ln x = \ln y + \ln c$

$\Rightarrow \ln x = \ln yc$

$\Rightarrow x = yc$

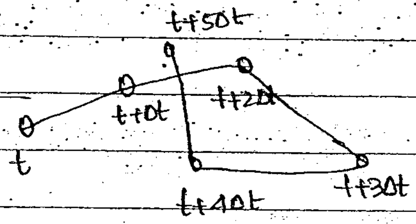
at $(1,2)$

$c = 1/2$

$\Rightarrow x = y/2 \Rightarrow 2x - y = 0$

Path Line

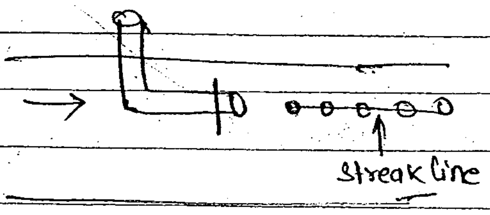
The path traced by a single fluid particles at different instant of time is known as a path line.

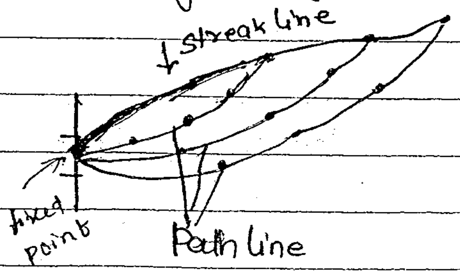


The path can cross each other.

Streak Line

It is the locus of various particles passing through a fixed point





for laminar flow, stream lines are constant.

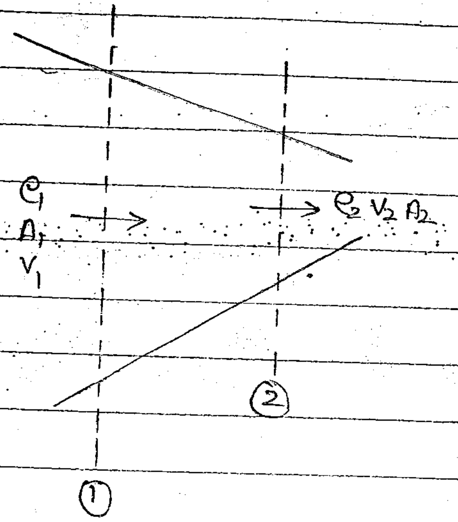
Continuity Eqⁿ (Mass Conservation)

Steady One-Dimensional Flow

m - kg

$\dot{m}$ - kg/sec = m/t

$\dot{m}$ entering = $\dot{m}$ leaving (for steady)



$\rho = \frac{m}{Vol} \Rightarrow m = \rho \times Vol$

$m = \rho \times A \times L$

$\dot{m} = \frac{m}{t} = \rho A \frac{L}{t}$

$\Rightarrow \dot{m} = \rho A V$

for steady flow mass entering is equal to mass leaving.

$\dot{m}_1 = \dot{m}_2$

$\Rightarrow \boxed{\rho_1 A_1 V_1 = \rho_2 A_2 V_2}$ Continuity eqⁿ for 1-D, steady flow.

$\boxed{\rho A V = \text{Constant}}$

for incompressible flow

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ $(\rho_1 = \rho_2)$ Continuity eqⁿ for 1-2,

$\boxed{A_1 V_1 = A_2 V_2}$ Steady and incompressible flow

Generalised Continuity Equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

(a) for steady flow $\frac{\partial \rho}{\partial t} = 0$

$$\Rightarrow \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

Continuity Eq^n in Cylindrical Coordinates

$$\frac{\partial (r v_r)}{\partial r} + \frac{\partial v_\theta}{\partial \theta} + r\frac{\partial v_z}{\partial z} = 0$$

(b). Incompressible flow

$$0 + \rho\frac{\partial u}{\partial x} + \rho\frac{\partial v}{\partial y} + \rho\frac{\partial w}{\partial z} = 0$$

$$\Rightarrow \rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] = 0$$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$ (for only incompressible flow).

This is valid for any incompressible flow.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$ for a 2-D incompressible flow

** Any fluid flow must satisfy Continuity eq^n, if the continuity eq^n is violated then such a flow is not possible.

Q54. A Compressible fluid is flowing steadily through a pipe whose area reduces by 40% from section 1 to section II. It is further known that corr. reduction in density is 15%. Compared to the velocity of fluid at section 1, the velocity at section 2 is increased by a factor of

(a) 2.96 (b) 1.96 (c) 3.96 (d) 4.96

Ans

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\Rightarrow \rho_2 = 0.85\rho_1$$

$$A_2 = 0.6 A_1$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$\Rightarrow \rho_1 A_1 V_1 = 0.85\rho_1 \times 0.6 A_1 \times V_2$$

$$\Rightarrow V_2 = 0.85 \times 0.6 V_2$$

$$\Rightarrow V_2 = 1.96 V_1$$

Q5~~4~~5 The velocity Components in x and y dirx^n is given by

$$u = \lambda x y^3 - x^2 y$$
$$v = x y^2 - \frac{3}{4} y^4$$

Then the value of λ for incompressible flow is

Ans- 2-D incompressible continuity eq^n is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \quad \frac{\partial u}{\partial x} = \lambda y^3 - x^2$$

$$\frac{\partial v}{\partial y} = 2xy - 3y^3$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \quad \lambda y^3 - 3y^3 = 0$$

$$\Rightarrow \lambda = 3.$$

Q56. The velocity of flow is given by

$$\vec{V} = (5x + 6y + 7z)\hat{i} + (6x + 5y + 9z)\hat{j} + (3x + 2y + \lambda z)\hat{k}$$

& $\rho = \rho_0 e^{-2t}$ (density variation)

in order that mass is conserved the Value of λ is

Ans. $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

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$$\frac{\partial \rho}{\partial t} = (-2)\,\rho_0 e^{-2t}$$

$$\frac{\partial \rho}{\partial t} = -2\rho_0 e^{-2t}$$

$$\Rightarrow \frac{\partial \rho}{\partial t} = -2\rho$$

as 'ρ' is notchanging with space

$$\Rightarrow -2\rho + \rho\frac{\partial u}{\partial x} + \rho\frac{\partial v}{\partial y} + \rho\frac{\partial w}{\partial z} = 0$$

$$\Rightarrow -2\cancel{\rho} + \cancel{\rho}(5) + \cancel{\rho}(5) + \lambda\cancel{\rho} = 0$$

$$\Rightarrow \lambda = \underline{\underline{-8}} \text{ Ans.}$$

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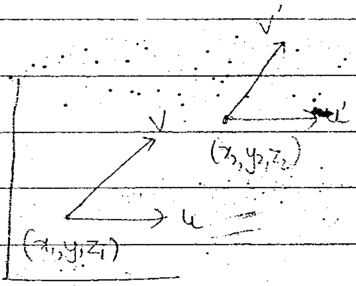
Acceleration of fluid particle

$\therefore V = u\hat{i} + v\hat{j} + w\hat{k}$



$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$|a| = \sqrt{a_x^2 + a_y^2 + a_z^2}$

so, Velocity at that point is thus a fxⁿ of its Coordinates

$a = \frac{dv}{dt} \Rightarrow a_x = \frac{du}{dt},\ a_y = \frac{dv}{dt}$

$a_z = \frac{dw}{dt}$

$u = f(x, y, z, t)$

$$\Rightarrow a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\frac{dx}{dt} + \frac{\partial u}{\partial y}\cdot\frac{dy}{dt} + \frac{\partial u}{\partial z}\cdot\frac{dz}{dt} + \frac{\partial u}{\partial t}\cdot\frac{\partial t}{\partial t}$$

$$\Rightarrow a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

$$a_x = \underbrace{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}_{\text{Convective}} + \underbrace{\frac{\partial u}{\partial t}}_{\text{Local or Temporal}}$$

Type of Flow	Convective	Local
Steady & Uniform	0	0
Steady & non-Uniform	Exists	0
Unsteady & Uniform	0	Exists
Unsteady & Non-Uniform	Exists	Exists

Discharge (θ)

Volume flow rate is known as discharge, it is also known as flow rate,

$$\theta = \frac{Vol}{time} = \frac{A \times L}{time} = A \times v$$

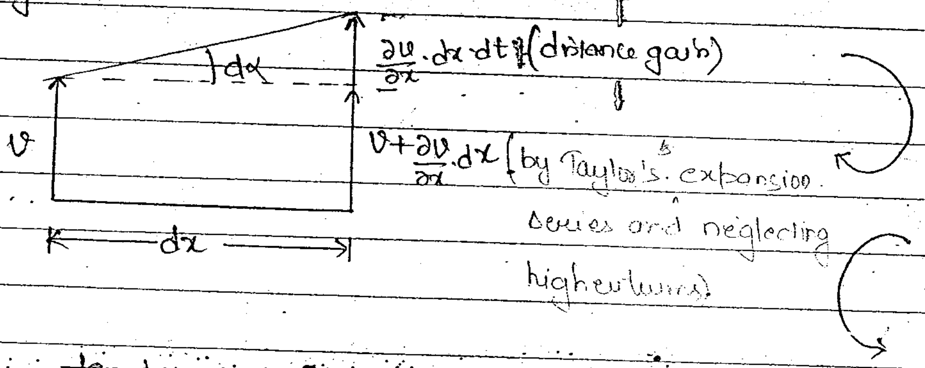
$$\Rightarrow \theta = Av$$

Discharge remains constant in a steady, 1-D, incompressible flow.

$$\left.\begin{array}{l} A_1V_1 = A_2V_2 \\ \theta_1 = \theta_2 \end{array}\right\} \begin{array}{l} \to \text{Steady} \\ \to \text{1-Dimensional} \\ \to \text{Incompressible} \end{array}$$

Rotational Component

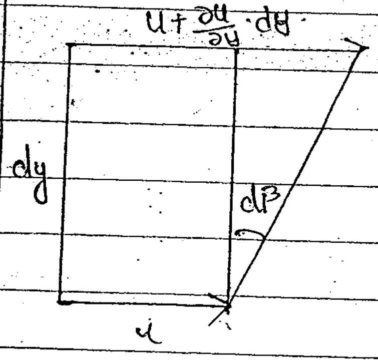
In fluid mechanics, the angular velocity ω is defined as the average angular velocity of initially two perpendicular line segments.



$$\tan d\alpha = \frac{\frac{\partial v}{\partial x} dx\, dt}{dx} = \frac{\partial v}{\partial x} dt$$

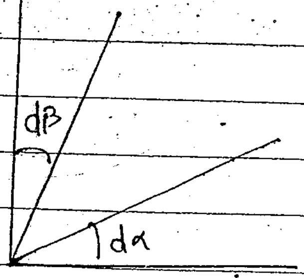
$$\tan d\alpha \approx d\alpha$$

$$\Rightarrow \frac{d\alpha}{dt} = \frac{\partial v}{\partial x} \quad (\text{anticlock} \Rightarrow +\text{ve})$$



$$\Rightarrow \frac{d\beta}{dt} = -\frac{\partial u}{\partial y}$$

↑ u decreasing in vertical dirxn due to viscosity.



$$\omega_z = \frac{1}{2}\left(\frac{d\alpha}{dt} + \frac{d\beta}{dt}\right)$$

$$\Rightarrow \omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\omega = \omega_x\hat{i} + \omega_y\hat{j} + \omega_z\hat{k}$$

$$\omega = \frac{1}{2}\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

Vorticity:- Twice of rotation is known as Vorticity.
A flow is said to be irrotational when $\omega_x = \omega_y = \omega_z = 0$

$$2\omega = \begin{vmatrix} i & \hat{j} & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

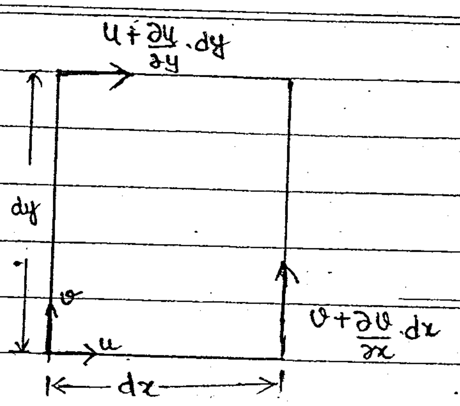
$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

*** Circulation (Γ):-

It is the line integral of the tangential Component of velocity taken around a Closed Curve.



$$\Gamma = u\,dx + \left(v+\frac{\partial v}{\partial x}dx\right)dy - \left(u+\frac{\partial u}{\partial y}dy\right)dx - v\,dy$$

$$\Rightarrow \Gamma = \cancel{u\,dx} + \cancel{v\,dy} + \frac{\partial u}{\partial x}dx\,dy - \cancel{u\,dx} - \frac{\partial u}{\partial y}dx\,dy - \cancel{v\,dy}$$

$$\Rightarrow \Gamma = \underbrace{\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)}_{\text{Vorticity}} dx\,dy$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

Circulation = Vorticity x Area

$$\Rightarrow 2\omega_z = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

In case of irrotational flow, $\omega = 0 \Rightarrow$ Vorticity $= 0$ and hence circulation is zero.

Every flow must satisfy continuity eqn.

Q57. The x component velocity in an incompressible 2-D flow is $u = 1.5x$. It is found that at a point (1,0) the y-component velocity is zero, then find the y-component velocity.

Ans. $u = 1.5x \Rightarrow \frac{\partial u}{\partial x} = 1.5$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow 1.5 + \frac{\partial v}{\partial y} = 0 \Rightarrow \frac{\partial v}{\partial y} = -1.5$$

$\Rightarrow \partial \vartheta = -1\cdot5\,\partial y$

$\Rightarrow \vartheta = -1\cdot5y + C$

$\Rightarrow 0 = -1\cdot5(0) + C$

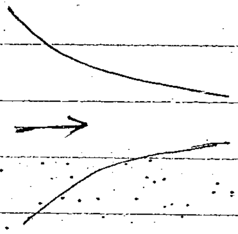
$\Rightarrow C = 0$

$\vartheta = -1\cdot5y$

Q58. A fluid with a discharge of 5m³ ~~cube~~ per second enters the nozzle as shown in fig. The cross-sectional area varies with x as $A = \frac{1}{1+x^2}$, assuming the flow to be steady one dimensional then find the acceleration at any point in nozzle.

Ans. $a = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

$a = a_x\hat{i}$ (1-dimensional)



$a = \sqrt{a_x^2} \Rightarrow a = a_x$

$a_x = u\frac{\partial u}{\partial x} + \cancel{v\frac{\partial u}{\partial y}}^{0} + \cancel{w\frac{\partial u}{\partial z}}^{0} + \cancel{\frac{\partial u}{\partial t}}^{0}$ [1-Dimensional & Steady]

$a = a_x = u\frac{\partial u}{\partial x}$

$Q = A \times V \Rightarrow V = \frac{Q}{A}$

$\Rightarrow a = a_x = \frac{Q}{A}\cdot\frac{\partial}{\partial x}\left(\frac{Q}{A}\right) = \frac{Q^2}{A}\cdot\frac{\partial}{\partial x}(1+x^2)$

$\Rightarrow a = \frac{Q^2}{A}(0+2x)$

$\Rightarrow a = 5^2 \times (1+x^2)(2x)$

$\Rightarrow a = 50x(1+x^2)$

$\Rightarrow a = 50(x+x^3)$ Ans.

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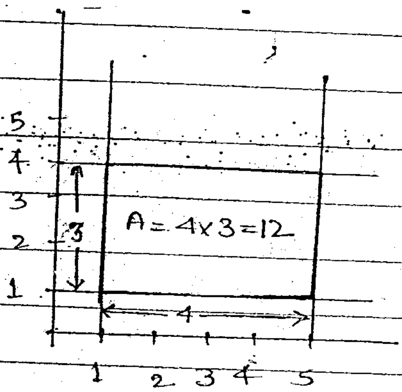
Q59. Determine the Circulation around a rectangle defined by $x=1, y=1, x=5, y=4$ for a velocity $u = 2x+3y$, $v = -2y$

Ans. $2\omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$



Vorticity = 0-3

= 0-3

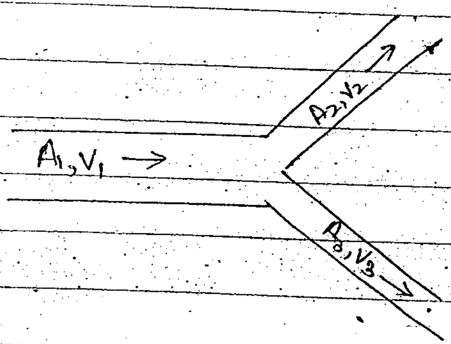
$\Rightarrow \Gamma = -3 \times 12 = -36$ Ans.

Q60 Water enters a pipe of Cross Sectional area A_1 that divides in to 2 Sections of equal areas A_2 and A_3, At one instant, the flow Velocities are $V_1 = 2$m/sec, $V_2 = 3$m/sec & $V_3 = 5$m/sec. At another instant $V_1 = 3$m/s, $V_2 = 4$m/s then find the value of V_3 at this instant.

Ans. I^st instant,

$A_2 = A_3$ (given)

$V_1 = 2, V_2 = 3, V_3 = 5$



$Q_1 = Q_2 + Q_3$

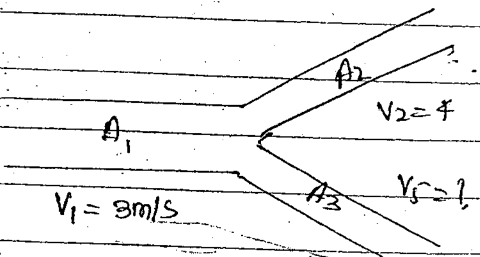
$\Rightarrow A_1V_1 = A_2V_2 + A_3V_3$

$\Rightarrow A \times 2 = A_2 \times 3 + A_2 \times 5$

$\Rightarrow 2A_1 = 8A_2 \Rightarrow A_1 = 4A_2$

II instant

$A_1V_1 = A_2V_2 + A_3V_3$

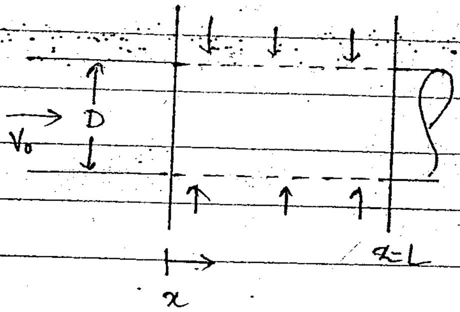


$\Rightarrow V_3 = 8$m/s

Q61. A pipe has a porous section of length 'L' as shown in fig. The velocity at the start of the section is V₀. If fluid leaks in to the pipe through porous section at a volumetric rate per unit area $\left(q\frac{x^2}{L^2}\right)$, What will be the velocity in pipe at any distance x, Assume 1-D incompressible flow.

Ans

$d\theta = \frac{qx^2}{L^2} \times \pi D \cdot dx$

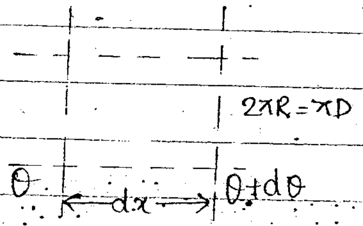


$\Rightarrow \theta = \frac{qx^3}{3L^2} \times \pi D + C$

At $x = 0 \Rightarrow \theta = \theta_0$

$\Rightarrow \theta_0 = 0 + C \Rightarrow C = \theta_0$

$\Rightarrow \theta = \frac{q\pi D}{L^2} \times \frac{x^3}{3} + \theta_0$



$\theta = AV$

$\Rightarrow V = \frac{\theta}{A}$

$\Rightarrow V = \frac{\frac{q\pi D}{L^2} \times \frac{x^3}{3} + \theta_0}{A}$

$\Rightarrow V = \frac{q\pi D x^3}{3L^2} \times \frac{1}{A} + \frac{\theta_0}{A}$

$\Rightarrow V = \frac{q\cancel{\pi}\cancel{D}x^3}{3L^2} \times \frac{1}{\frac{\cancel{\pi}}{4}D^2} + V_0$

$= \frac{4qx^3}{3L^2D} + V_0$

Q62 The Velocity for a 2-D flow is as follows

$u = \frac{U_0 x}{L}$; $v = -\frac{U_0 y}{L}$, If $L = 0.2$, and the resultant of total acceleration in x & y dirn's at $x = L$, $y = L$ ~~is equal to~~ is $10\,m/s^2$, (i) Then value of U_0 in m/s is

(a) 1.414 (b) 2.38 (c) 1.19 (d) 11.9

(ii). This fluid is

(a). Rotational & Compressible

(b). irrotational & Compressible

(c). Rotational & Incompressible

(d). Irrotational & Incompressible

Ans. (ii) $u = \frac{x u_0}{L}$, $v = -\frac{u_0 y}{L}$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ → incompressible

⇒ $\frac{\partial u}{\partial x} = \frac{U_0}{L}$; $\frac{\partial v}{\partial y} = -\frac{U_0}{L}$

⇒ $\frac{U_0}{L} - \frac{U_0}{L} = 0$ ⇒ flow is incompressible.

$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

⇒ $\omega_z = \frac{1}{2}(0-0) = 0$ ⇒ irrotational

(i) $a = a_x\hat{i} + a_y\hat{j}$ $a = 10$

$a = \sqrt{a_x^2 + a_y^2}$ ⇒ $a_x^2 + a_y^2 = 100$ —— ①

$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$

$a_x = \frac{U_0 x}{L} \times \frac{U_0}{L} + \left(-\frac{V_0 y}{L}\right)(0) + 0$

⇒ $a_x = \frac{U_0^2 x}{L^2}$

$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \frac{\partial v}{\partial t}$

⇒ $a_y = \frac{U_0 x}{L}(0) + \left(-\frac{U_0 y}{L}\right)\left(-\frac{U_0}{L}\right) + 0$

⇒ $a_y = \frac{U_0^2 y}{L^2}$

from ① $a_x^2 + a_y^2 = 100$

⇒ $\left(\frac{U_0^2 x}{L^2}\right)^2 + \left(\frac{U_0^2 y}{L^2}\right)^2 = 100$

⇒ $\left(\frac{U_0^2}{L^2}\right)^2 (x^2+y^2) = 100$

⇒ $\left(\frac{U_0^2}{L^2}\right)^2 = \frac{100}{x^2+y^2}$

⇒ $\frac{U_0^2}{L^2} = \sqrt{\frac{100}{x^2+y^2}}$

⇒ $\frac{U_0^2}{L^2} = \sqrt{\frac{100}{L^2+L^2}} = \frac{\sqrt{100}}{\sqrt{2}\cancel{L}}$

⇒ $\frac{U_0^2}{L} = \frac{10}{\sqrt{2}}$ ⇒ $U_0^2 = \frac{10}{\sqrt{2}} \times 0.2$

⇒ $U_0^2 = 1.414$

⇒ $U_0 = \underline{1.19}$ Ans.

Velocity Potential fx^n (ϕ)

It is a fx^n of space and time defined in such a manner that its negative derivative with space will give velocity in that dirx^n. The -ve sign is taken because the flow is in the dirx^n of decreasing potential.

Velocity potential fx^n can be ~~deteum~~ defined for 3-D flow.

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$$-\frac{\partial \phi}{\partial x} = u$$

$$-\frac{\partial \phi}{\partial y} = v$$

$$-\frac{\partial \phi}{\partial z} = w$$

~~for~~ for 2-D, incompressible, steady flow

⇒ Continuity eqⁿ is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial y}\right)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\left[\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2}\right]$$

Case(i) If ϕ satisfies laplace eqⁿ ⇒ $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial \phi}{\partial y^2} = 0$

$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ ⇒ Continuity eqⁿ is satisfied & flow is possible

Case(ii) If ϕ doesnot satisfy Laplace eqⁿ

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} \neq 0$$

$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \neq 0$ ⇒ Continuity eqⁿ is not satisfied flow is not possible.

A flow is possible only when the ~~the~~ velocity potential fxⁿ satisfies Laplace eqⁿ.

$$u = -\frac{\partial \phi}{\partial x} \quad , \quad v = -\frac{\partial \phi}{\partial y}$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial}{\partial x}\left(-\frac{\partial \phi}{\partial y}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \phi}{\partial x}\right)\right]$$

$$\Rightarrow \omega_z = \frac{1}{2}\left[-\frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y \partial x}\right]$$

$$\Rightarrow \omega_z = 0 \Rightarrow \text{Irrotational}$$

** Velocity potential fx^n exists only for irrotational flow, i.e. the existence of velocity potential fx^n implies that the flow is irrotational.

Stream function ~~(ϕ)~~ (ψ) :-

It is a fx^n of space & time defined in such a manner that it satisfies the continuity eq^n.

$$u = -\frac{\partial \psi}{\partial y} , \quad v = \frac{\partial \psi}{\partial x}$$

We can also define by foreign authors

$$u = \frac{\partial \psi}{\partial y} \quad v = -\frac{\partial \psi}{\partial x}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial}{\partial x}\left(-\frac{\partial \psi}{\partial y}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial x}\right)$$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{\partial^2 \psi}{\partial x \partial y} + \frac{\partial^2 \psi}{\partial y \partial x}$$

$$\Rightarrow \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Sum of partial derivative is total derivative.

3|1|2011

$$u = -\frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \psi}{\partial x}$$

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

$$\Rightarrow W_z = \frac{1}{2}\left(\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(\frac{-\partial \psi}{\partial y}\right)\right)$$

$$\Rightarrow \omega_z = \frac{1}{2}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right)$$

Case(i). if ψ satisfies laplace eqn, $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0$

$\omega_z = \frac{1}{2}(0) = 0 \Rightarrow \omega_z = 0 \Rightarrow$ irrotational

Case(ii). If ψ doesnot satisfy laplace eqn $\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \neq 0$

$\Rightarrow \omega_z \neq 0 \Rightarrow$ Rotational.

Velocity potential fxn exists only for irrotational flow but stream fxn exists for rotational and irrotational flow. If the stream function satisfies Laplace eqn then the flow is irrotational, otherwise the flow is rotational.

~~Udx~~

$$vdx - udy = 0 \rightarrow \text{Eq}^n \text{ of streamline}$$

$$\frac{\partial \psi}{\partial x} \cdot dx - \left(\frac{-\partial \psi}{\partial y}\right) dy = 0$$

$$\Rightarrow \frac{\partial \psi}{\partial x} \cdot dx + \frac{\partial \psi}{\partial y} \cdot dy = 0 \quad \text{——} \quad ①$$

$\psi(x,y,t)$ i-e ~~$\psi f(x$~~ $\psi = f(x,y,t)$

At any given instant, $\psi = f(x,y)$

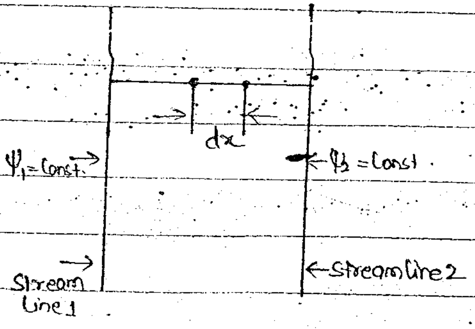
$\Rightarrow \; d\psi = \frac{\partial \psi}{\partial x} dx. + \frac{\partial \psi}{\partial y} . dy$ —— ②

for a particular stream line ① & ②

$d\psi = 0 \Rightarrow \psi = \text{Constant}$

*** for a particular stream line, stream fxn remains constant

Significance of stream fxn



$\theta = AV$

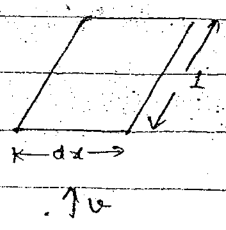
$\Rightarrow v = \frac{\partial \psi}{\partial x}$

$\Rightarrow d\theta = v \times dx \times 1$

$\Rightarrow d\theta = \frac{\partial \psi}{\partial x} . dx$ —— ①

as $\partial \psi = \frac{\partial \psi}{\partial x} . dx + \frac{\partial \psi}{\partial y} . dy$

Since no change in y-dirxn



$\Rightarrow d\psi = \frac{\partial \psi}{\partial x} dx$ —— ②

from ① and ②

$d\theta = d\psi$

*** The difference b/w any two stream fxn's will give discharge

Unit width.

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Relationship b/w Equipotential lines and Constant Stream fxn Lines

$$\phi(x,y) = \text{Constant}$$

$$d\phi = \frac{\partial \phi}{\partial x}.dx + \frac{\partial \phi}{\partial y}.dy = 0$$

$$\frac{\partial \phi}{\partial x}.dx = -\frac{\partial \phi}{\partial y}.dy \qquad u = \frac{\partial \phi}{\partial x}, \quad v = -\frac{\partial \psi}{\partial y}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\partial \phi / \partial x}{-\partial \phi / \partial y}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = -\frac{u}{v}} \rightarrow \text{Slope of equipotential line}$$

$$\psi(x,y) = \text{Const}$$

$$d\psi = \frac{\partial \psi}{\partial x}.dx + \frac{\partial \psi}{\partial y}.dy = 0$$

$$u = -\frac{\partial \psi}{\partial y}, \quad v = \frac{\partial \psi}{\partial x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\partial \psi / \partial x}{-\partial \psi / \partial y}$$

$$\Rightarrow \boxed{\frac{dy}{dx} = \frac{v}{u}} \rightarrow \text{Slope of Constant stream fxn.}$$

← Eqn of Streamline

$$\text{Product of slopes} = -\frac{u}{v} \times \frac{v}{u} = -1$$

Equipotential lines & Constant stream fxn lines are Orthogonal (perpendicular) to each other in flow-field.

Cauchy - Reimann (C-R) eq^ns

$$u = -\frac{\partial \phi}{\partial x} = \frac{-\partial \psi}{\partial y}$$

$$\Rightarrow \boxed{\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}}$$

$$v = -\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$$

$$\Rightarrow \boxed{-\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}}$$

Q63/ The stream fx^n is given by $\psi = 3xy$, then find the velocity at (2,3).

Ans $\psi = 3xy$

$$V = u\hat{i} + v\hat{j}$$

$$V = \sqrt{u^2+v^2}$$

$$u = -\frac{\partial \psi}{\partial y} = -3x$$

$$v = \frac{\partial \psi}{\partial x} \Rightarrow v = 3y$$

at (2,3) $u = -6, \quad v = 9$

$$\Rightarrow V = \sqrt{(-6)^2 + (9)^2}$$

Q64/ The stream fx^n for a 2-D stream line flow is given by $\psi = Px^2 + Qy^2$ the velocity potential fx^n for this flow exists only when

(a) $P = Q$
(b) $P = -Q$
(c) $P = Q/2$
(d) $P = 2Q$

Ans Velocity potential will exist only if the flow is irrotational and when flow is irrotational it has to satisfy Laplace eqn.

$$\psi = Px^2 + Qy^2$$

$$\Rightarrow \quad \frac{\partial^2\phi}{\partial y^2} = Q \quad \text{and} \quad \frac{\partial^2\phi}{\partial x^2} = P$$

by Laplace eqn

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0$$

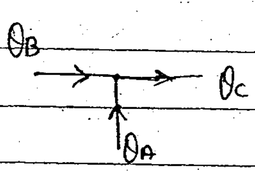
$$\Rightarrow \quad P + Q = 0 \quad \Rightarrow P = -Q$$

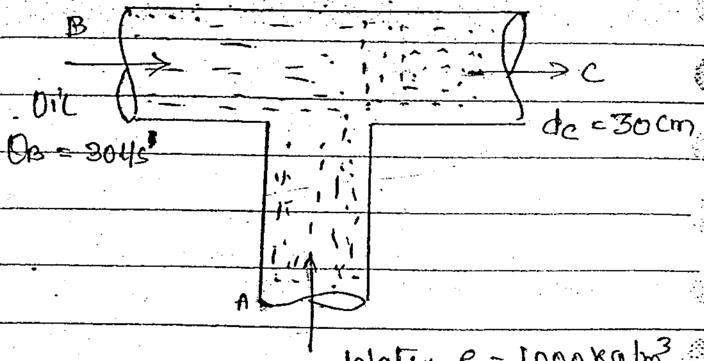
***

Q65
G.A.T.E-2012

Water enters a mixing device steadily at 150L/sec through pipe A, while oil with density 800kg/m³ is forced in at 30L/sec through pipe B. If the liquids are incompressible and forms a homogeneous mixture of oil & water find the velocity and density of the mixture leaving through a pipe 'C' of diameter 30cm.

Ans





$Q_A + Q_B = Q_c$

$Q_c = 180$ l/sec

1 litre $= 10^{-3} m^3$

Water $\rho = 1000 kg/m^3$

$\Rightarrow Q_c = 180 \times 10^{-3} \; m^3/sec$

$\phi = AV$

$\theta_c = \frac{\pi}{4} d_c^2 \times V_c$

$\Rightarrow 180\times10^{-3} = \frac{\pi}{4} . (0\cdot3)^2 \times V_c$

$\Rightarrow V_c = 2\cdot54 m/s$

$\dot{m}_A + \dot{m}_B = \dot{m}_C$

$\rho_A \theta_A + \rho_B \theta_B = \rho_c \theta_c$

$10^3 \times 150 + 800 \times 30 = \rho_c \times 180$

$\Rightarrow \rho_c = 966\cdot6$ kg/m³.

Q66. For a 2-D irrotational flow the velocity potential fxn is defined as $\phi = \ln(x^2+y^2)$ then which of the following is a possible stream function.

(a) $\frac{1}{2}\tan^{-1}\left(\frac{y}{x}\right)$, (b) $\tan^{-1}(y/x)$ (c) $2\tan^{-1}\left(\frac{y}{x}\right)$ (d) $2\tan^{-1}\left(\frac{x}{y}\right)$

Ans. $\phi = \ln(x^2+y^2)$

$\Rightarrow \frac{\partial\phi}{\partial x} = \frac{2x}{x^2+y^2}$

$\frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y}$

$\Rightarrow \frac{\partial\psi}{\partial y} = \frac{2x}{x^2+y^2}$ $\Rightarrow \frac{\partial\psi}{\partial y} = \frac{2x}{x^2[1+y^2/x^2]}$

$\Rightarrow \frac{\partial\psi}{\partial y} = \frac{2}{x[1+(\frac{y}{x})^2]}$

$\Rightarrow \partial\psi = \frac{2\,\cancel{\partial\psi}\partial y}{x[1+(\frac{y}{x})^2]}$ $\Rightarrow \psi = \int \frac{2\,\partial\cancel{\psi}y}{x[1+(y/x)^2]}$

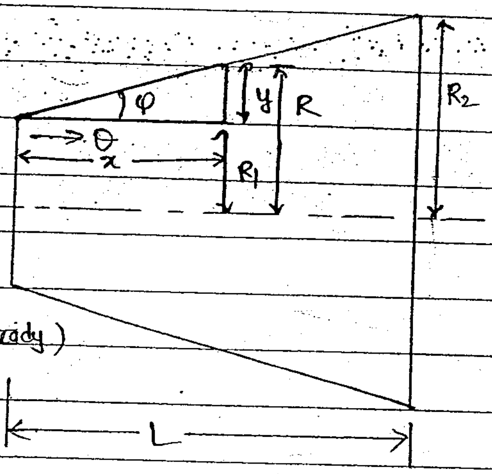
$\Rightarrow \psi = 2\tan^{-1}(y/x)$

Q67. For a liquid flowing through a divergent passage having inlet and outlet radii (R_1 & R_2) and a constant flow rate of θ, find the acceleration at the exit of pipe, assume the flow to be steady one dimensional incompressible.

Ans. As its 1-D

$$\Rightarrow a = a_x\hat{i} + \overset{0}{\cancel{a_y}}\hat{j} + \overset{0}{\cancel{a_z}}\hat{k}$$

$$\Rightarrow a = a_x$$



and $a_x = u\frac{\partial u}{\partial x} + \overset{0}{\cancel{v\frac{\partial u}{\partial y}}} + \overset{0}{\cancel{w\frac{\partial u}{\partial z}}} + \cancel{\frac{\partial u}{\partial t}}$ (steady)

$$a_x = a = u\frac{\partial u}{\partial x}.$$

$$V = u\hat{i} + \overset{0}{\cancel{v}}\hat{j} + \overset{0}{\cancel{w}}\hat{k} \Rightarrow V = u,$$

$$a = V\frac{\partial V}{\partial x}$$

$$\theta = A \times V \quad \Rightarrow V = \frac{\theta}{A}$$

(In steady flow one dimensional incompressible flow discharge is constant.)

$$\Rightarrow a = \frac{\theta}{A}\frac{\partial}{\partial x}\left(\frac{\theta}{A}\right)$$

$$\Rightarrow a = \frac{\theta^2}{A}\frac{\partial}{\partial x}\left(\frac{1}{A}\right)$$

Now, $R = R_1 + y$

$\tan\theta = y/x$

$\Rightarrow y = x\tan\theta$

$\Rightarrow R = R_1 + x\tan\varphi$

$\tan\varphi = \frac{R_2 - R_1}{L}$ = Constant (K)

$\Rightarrow R = R_1 + xK.$

$\Rightarrow A = \pi R^2.$

$A = \pi(R_1 + Kx)^2$

$$a = \frac{\theta^2}{A} \cdot \frac{\partial}{\partial x}\left(\frac{1}{A}\right)$$

$$a = \frac{\theta^2}{\pi(R_1+kx)^2} \cdot \frac{\partial}{\partial x}\left(\frac{1}{\pi(R_1+kx)^2}\right)$$

$$a = \frac{\theta^2}{\pi^2(R_1+kx)^2} \cdot \frac{\partial}{\partial x} \times \frac{1}{(R_1+kx)^2}$$

$$a = \frac{\theta^2}{\pi^2(R_1+kx)^2}\left[\frac{-2}{(R_1+kx)^3}(0+k(1))\right]$$

$$a = \frac{-2k\theta^2}{\pi^2(R_1+kx)^5}$$

$$a = \frac{-2\theta^2(R_2-R_1)}{\pi^2 L(R_1+kx)^5}$$

at exit $x = L$

$$a = \frac{2\theta^2(R_1-R_2)}{\pi^2 L(R_1+kL)^5}$$

$$\Rightarrow a = \frac{2\theta^2(R_1-R_2)}{\pi^2 L R_2^5}$$

Ans.

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FLUID DYNAMICS

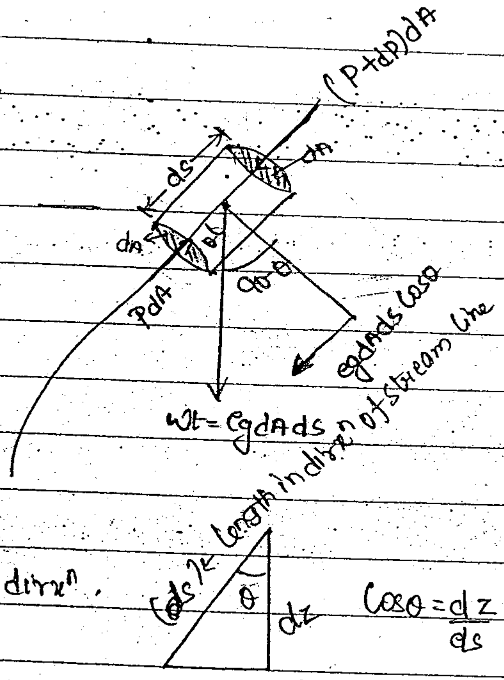
Laminar flow is viscous flow.

Generally, forces acting on fluid element are pressure force F_p, Gravity force F_g & viscous force F_v. In Navier Stoke's analysis pressure, gravity and viscous forces are taken in consideration.

If viscous forces are neglected then the forces acting are gravity and pressure forces in Euler's analysis.

Bernoulli's Eqn (Conservation of Energy)

Assumptions
- The flow is non viscous.
- Steady flow
- Incompressible flow.



$\cos\theta = \frac{dz}{ds}$

$W = mg \Rightarrow m = \frac{W}{g}$

$\Rightarrow m = \rho dA ds$

$a_s = \frac{V\partial V}{\partial s} + \frac{\partial V}{\partial t}$

$a_s \rightarrow$ acc. in dirxn of Stream line dirxn.

$\Sigma F = ma$

$\Rightarrow PdA - (P+dP)\,d\cancel{A} - \rho g\,d\cancel{A}\,ds\cos\theta = \rho\,d\cancel{A}\,ds \times \left[\frac{V\partial V}{\partial s} + \frac{\partial V}{\partial t}\right]$

$\Rightarrow -dP - \rho g\,dz = \rho ds\left[\frac{V\partial V}{\partial s} + \frac{\partial V}{\partial t}\right]$ —— I (Euler's Eqn) for steady flow $\frac{\partial v}{\partial t} = 0$

$$\Rightarrow -dp - \rho g\,dz = \rho ds\left[\frac{v\partial v}{\partial s}\right] = \rho \cancel{ds}\left(\frac{v\,dv}{\cancel{ds}}\right)$$

$$\Rightarrow \quad -dp - \rho g dz = \rho v dv$$

$$\Rightarrow \quad dp + \rho g dz + \rho v dv = 0$$

$$\Rightarrow \quad \boxed{\frac{dp}{\rho} + g dz + v dv = 0} \leftarrow \text{Euler's Eq}^n \text{ for Steady flow}$$

↑ momentum Eqn

for Incompressible flow,

$$\int \frac{dp}{\rho} + \int g dz + \int v dv = \text{Constant}$$

$$\Rightarrow \boxed{\frac{P}{\rho} + gz + \frac{V^2}{2} = \text{Constant}} \leftarrow \text{Bernoulli's Eq}^n.$$

In this equation each term represents energy of the fluid per unit mass according to Bernoulli's eqn in a steady, non-viscous incompressible flow the sum of pressure energy, kinetic energy and potential energy is constant and hence Bernoulli's eqn is mechanical energy balance eqn.

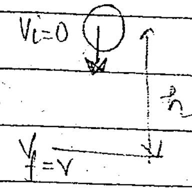
Energy/mass $\Rightarrow \frac{P}{\rho} + gz + \frac{V^2}{2} = \text{Const.}$

Energy/mass × g $\Rightarrow \frac{P}{\rho g} + z + \frac{V^2}{2g} = \text{Constant}$

$$\Rightarrow \frac{\text{Energy}}{\text{weight}} = \frac{P}{w} + \frac{V^2}{2g} + z = \text{Const.}$$

$\frac{P}{w}$ = Pressure head

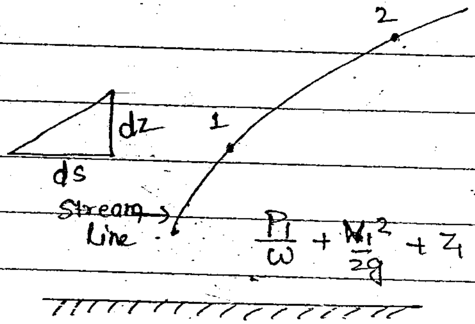
$\frac{V^2}{2g}$ → kinetic head or Dynamic head →



Z = P.E. head

$\frac{P}{w} + Z$ = Piezometric head.

$\frac{P}{w} + \frac{V^2}{2g} + z$ = Total head.



$$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2$$

Relationship b/w 1st Law of T.D and Bernoulli's eqn:-

$$h_1 + \frac{V_1^2}{2} + Z_1 g + \theta = h_2 + \frac{V_2^2}{2} + Z_2 g + W$$

$$U_1 + P_1 v_1 + \frac{V_1^2}{2} + Z_1 g + \theta = U_2 + P_2 v_2 + \frac{V_2^2}{2} + Z_2 g + W$$

$$U_1 + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} + Zg + \theta = U_2 + \frac{P_2}{\rho_2} + \frac{V_2^2}{2} + Zg + W$$

$$\Rightarrow \frac{P_1}{\rho} + \frac{V_1^2}{2} + Z_1 g = \frac{P_2}{\rho} + \frac{V_2^2}{2} + Z_2 g$$

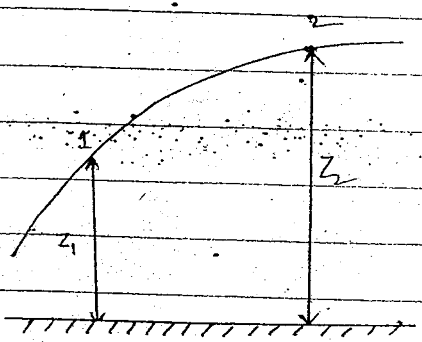
Assumptions
- Steady flow
- $U_1 = U_2$
- $\theta = 0$
- $W = 0$
- Incompressible $(\rho_1 = \rho_2)$

$$\Rightarrow \frac{P_1}{\rho g}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+Z_2$$

4|1|2012 **Bernoulli's Eqn for a real fluid flow problem :-**

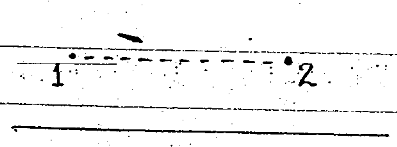
$$\frac{P_1}{w}+\frac{V_1^2}{2g}+Z_1=\frac{P_2}{w}+\frac{V_2^2}{2g}+Z_2+h_L$$

$h_L \rightarrow$ head loss by viscosity :



Q68. At a location 1 of horizontal pipe the pressure head is 32cm. and Velocity head is 4cm. the reduction in area at location 2 is such that the pressure head drops down to zero then find the ratio of velocity at Section 2 to Section 1.

Ans. $\frac{P_1}{w}=32, \quad \frac{V_1^2}{2g}=4$



$\frac{P_2}{w}=0, \quad \frac{V_2^2}{2g}=36$

$$\frac{P_1}{w}+\frac{V_1^2}{2g}=\frac{P_2}{w}+\frac{V_2^2}{2g}$$

$$\Rightarrow 32+4=0+\frac{V_2^2}{2g}$$

$$\Rightarrow \frac{V_2^2}{2g}=36 \quad \Rightarrow \frac{V_1^2}{2g}=4$$

$$\Rightarrow \frac{V_2^2}{V_1^2}=9 \quad \Rightarrow \frac{V_2}{V_1}=3$$

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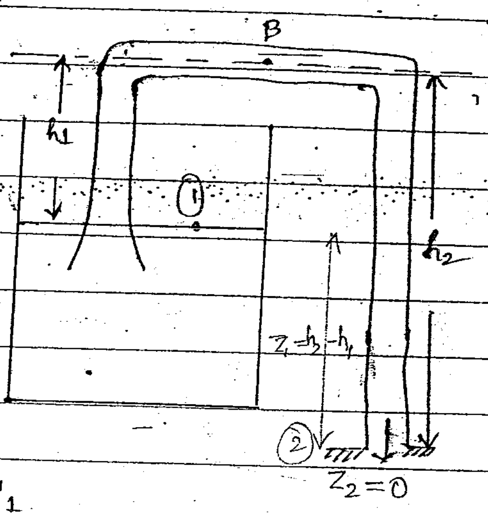
Q69 A pipe draws ^water^ from a reservoir at discharges it out at atmospheric pressure, Assuming ideal fluid and reservoir is very large. find the velocity at point 'B' in pipe :

Ans. $P_1 = P_2 = P_{atm}$



$$\frac{\cancel{P_1}}{\omega} + \frac{V_1^2}{2g} + Z_1 = \frac{\cancel{P_2}}{\omega} + \frac{V_2^2}{2g} + Z_2$$

$A_1V_1 = A_2V_2$

A_1 is large $\Rightarrow V_1 = \frac{A_2V_2}{A_1}$

$\Rightarrow$ V_1 is small so we can neglect V_1

① and ② की information ज्ञात so using this.

$$\Rightarrow \cancel{\frac{V_1^2}{2g}} + (h_2 - h_1) = \frac{V_2^2}{2g} + 0$$

$$\Rightarrow V_2 = \sqrt{2g(h_2 - h_1)}$$

as $A_B V_B = A_2 V_2$ (by continuity eqn)

~~$\Rightarrow$~~ $A_B = A_2$

$\Rightarrow V_B = V_2$

$$\Rightarrow V_B = V_2 = \sqrt{2g(h_2 - h_1)}$$

***

Q70. A free jet of water is emerging from a nozzle of 75mm diameter attached to a pipe 225 mm diameter as shown in fig. The velocity of water at point 'A' is 18m/s, neglecting friction in pipe and nozzle find the velocity ^of^ water at the nozzle tip. Also find pressure at point B in kPa.

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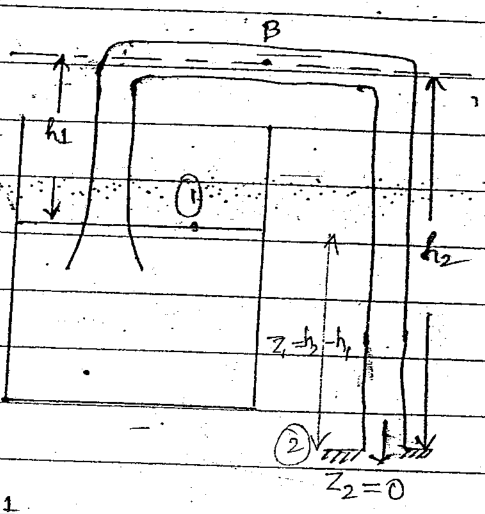
Q69 A pipe draws Water from a reservoir at discharges it out at atmospheric pressure, Assuming ideal fluid and reservoir is very large. find the velocity at point 'B' in pipe :

Ans. $P_1 = P_2 = P_{atm}$



① and ② પે information આપેલ so using this.

$$\frac{\cancel{P_1}}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{\cancel{P_2}}{w} + \frac{V_2^2}{2g} + Z_2$$

$$A_1 V_1 = A_2 V_2$$

A_1 is large $\Rightarrow V_1 = \frac{A_2 V_2}{A_1}$

$\Rightarrow V_1$ is small so we can neglect V_1

$$\Rightarrow \cancel{\frac{V_1^2}{2g}} + (h_2 - h_1) = \frac{V_2^2}{2g} + 0$$

$$\Rightarrow V_2 = \sqrt{2g(h_2 - h_1)}$$

as $A_B V_B = A_2 V_2$ (by Continuity eqⁿ)

~~⇒~~ $A_B = A_2$

$\Rightarrow V_B = V_2$

$$\Rightarrow V_B = V_2 = \sqrt{2g(h_2 - h_1)}$$

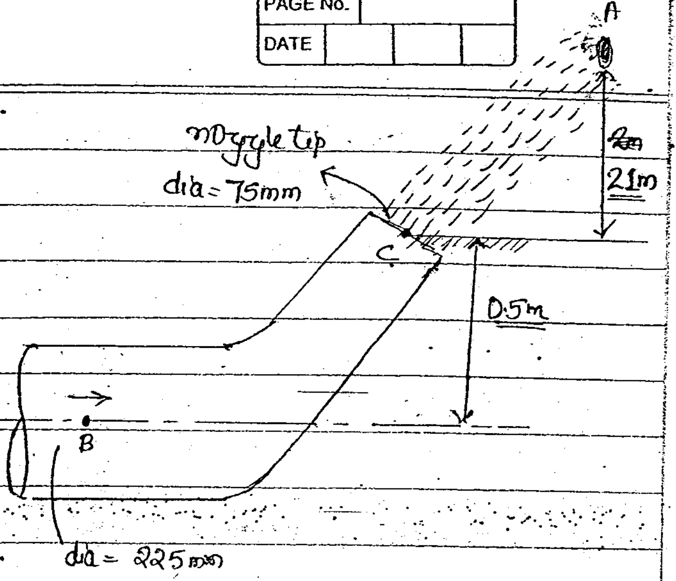
***

Q70. A free jet of Water is emerging from a nozzle of 75mm diameter attached to a pipe 225 mm diameter as shown in fig. The velocity of Water at point 'A' is 18m/s, neglecting friction in pipe and nozzle find the Velocity of Water at the nozzle tip. Also find pressure at point B in Kpa.

Ans.

$P_c = P_A = P_{atm}$

$\frac{\cancel{P_c}}{\cancel{w}} + \frac{V_c^2}{2g} + Z_c = \frac{\cancel{P_A}}{\cancel{W}} + \frac{V_A^2}{2g} + Z_A$



$\Rightarrow \frac{V_c^2}{2g} + Z_C = \frac{V_A^2}{2g} + Z_A$

$\Rightarrow \frac{V_c^2}{2 \times 9.81} + 0 = \frac{18^2}{2 \times 9.81} + 21$

$\Rightarrow \boxed{V_c = 27.1 m/s}$

Continuity eqn b/w B & C,

$A_B V_B = A_C V_c$

$\frac{\cancel{\pi}}{\cancel{4}} d_B^2 \times V_B = \frac{\cancel{\pi}}{\cancel{4}} d_c^2 \times V_c$

$\Rightarrow 225^2 \times V_B = 75^2 \times 27.1$

$V_B = 3.01$ m/s

$P_c = P_{atm} = 0$ (as we calculate gauge pressures only):-

$\frac{P_B}{w} + \frac{V_B^2}{2g} + Z_B = \frac{P_c}{W} + \frac{V_c^2}{2g} + Z_c$

$\Rightarrow \frac{P_B}{9810} + \frac{3.01^2}{2 \times 9.81} + 0 = \frac{27.1^2}{2 \times 9.81} + 0.5$

$\Rightarrow P_B = 368.3 kN/m^2$

****

Q71. Consider Euler's eqn for 1-D horizontal unsteady flow, the diameter of Pipe is 20cm Water discharge increases from 25 to 100 litres per second in 3 seconds what is the pressure gradient that can sustain the flow per metre length.

Ans. Discharge is not varying with space but is varying with time. –
Discharge is constant with space.

Pipe is horizontal

$\Rightarrow dz=0$

$Q = AV$

$\Rightarrow$ V is also constant with space.

$$\Rightarrow \frac{\partial V}{\partial s} = 0$$

$$\Rightarrow -dp - \rho g \, \overset{0}{\cancel{dz}} = \rho ds \left[V \overset{0}{\cancel{\frac{\partial V}{\partial s}}} + \frac{\partial V}{\partial t} \right]$$

horizontal

discharge is not varying with space

$$\Rightarrow -dp = \rho ds \left[\frac{\partial V}{\partial t} \right]$$

$$\frac{dP}{ds} = -\rho \left[\frac{\partial V}{\partial t} \right]$$

$$\Rightarrow \frac{dP}{ds} = -\rho \left[\frac{\partial}{\partial t} \left(\frac{Q}{A} \right) \right]$$

$$\Rightarrow \frac{dp}{ds} = -\frac{\rho}{A} \frac{\partial Q}{\partial t}$$

$$\Rightarrow \frac{dP}{ds} = \frac{-10^3 \times 75 \times 10^{-3}}{\frac{\pi}{4}(0.2)^2 \ 3}$$

$$\Rightarrow \frac{dP}{ds} = -795.7 \text{ Pa/m}$$

700

$\theta_1 = 25$

$\theta_2 = 100$

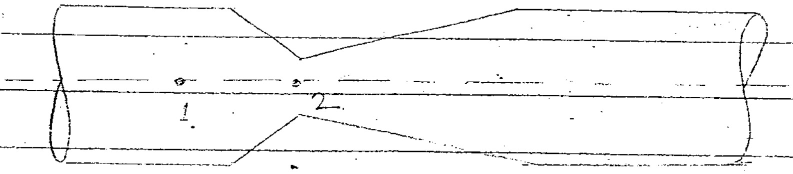
$t = 3$

$\Rightarrow \frac{d\theta}{dt} = \frac{100-25}{3}$

$\Rightarrow \frac{d\theta}{dt} = \frac{75}{3} \times 10^{-3} = 25\times10^{-3}$

Applications of Bernoulli's Equation :-

○ **Venturimeter** :- It is used for finding out discharge.



$$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2$$

$$\left(\frac{P_1}{w} + Z_1\right) - \left(\frac{P_2}{w} + Z_2\right) = \frac{V_2^2 - V_1^2}{2g} = h$$

horizontal $\Rightarrow Z_1 = Z_2$

$$\frac{P_1}{w} - \frac{P_2}{w} = \frac{V_2^2 - V_1^2}{2g} = h$$

$$\Rightarrow V_2^2 - V_1^2 = 2gh$$

$$\theta = a_1V_1 = a_2V_2$$

$$\Rightarrow V_1 = \frac{\theta}{a_1} ; V_2 = \frac{\theta}{a_2}$$

$$\Rightarrow \frac{\theta^2}{a_2^2} - \frac{\theta^2}{a_1^2} = 2gh$$

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$$\theta^2\left[\frac{1}{a_2^2}-\frac{1}{a_1^2}\right]=2gh$$

h → horizontal pressure head diff.

If inclined

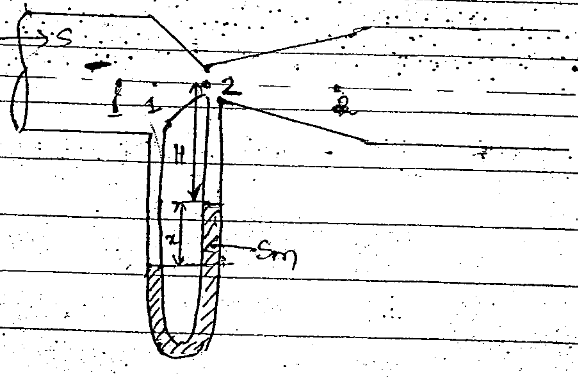
h → piezometric head diff

$$\theta^2\left[\frac{a_1^2-a_2^2}{a_1^2a_2^2}\right]=2gh$$

$$\theta=\frac{a_1a_2\sqrt{2gh}}{\sqrt{a_1^2-a_2^2}}$$

While applying Bernoulli's Eqn as no losses were taken into consideration ∴ its theoretical discharge

$$\therefore\ \theta_{th}=\left(\frac{a_1a_2\sqrt{2gh}}{\sqrt{a_1^2-a_2^2}}\right)$$



$P_1 > P_2$

$a_1V_1 = a_2V_2$

$a_2 < a_1 \Rightarrow V_2 > V_1$

$$\Rightarrow \frac{P_1}{w}+\frac{V_1^2}{2g}=\frac{P_2}{w}+\frac{V_2^2}{2g}$$

$\Rightarrow \quad P_2 < P_1$

$$\frac{P_1}{w} + H + x - \frac{x S_m}{S} - H = \frac{P_2}{w}$$

$$\Rightarrow \frac{P_1 - P_2}{w} = \frac{x S_m}{S} - x$$

$$\Rightarrow \frac{P_1 - P_2}{w} = x\left(\frac{S_m}{S} - 1\right)$$

$$\Rightarrow \frac{P_1 - P_2}{w} = h = x\left(\frac{S_m}{S} - 1\right)$$

Principle of Venturimeter

By decreasing area in a steady 1-D incompressible flow velocity increases, this increase in velocity results in decrease in pressure, and due to this pressure ~~t~~ difference there is a manometric fluid deflection when differential manometer is connected and by measuring this deflection discharge can be found.

Coefficient of Discharge (Cd)

It is the ~~Cd~~ ratio of actual discharge to the theoretical discharge. As venturimeter is a gradually converging and diverging device losses are less and Cd is 0.94 to 0.98.

$$Cd = \frac{Q_{act}}{Q_{th}}$$

$$\Rightarrow Q_{act} = \frac{Cd\, a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

When there is head loss

$$\frac{P_1}{W} + \frac{V_1^2}{2g} = \frac{P_2}{W} + \frac{V_2^2}{2g} + h_L$$

$$\Rightarrow \left(\frac{P_1}{w} - \frac{P_2}{w}\right) - h_L = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$\Rightarrow h - h_L = \frac{V_2^2 - V_1^2}{2g}$$

$$\Rightarrow V_2^2 - V_1^2 = 2g(h - h_L)$$

$$Q = a_1V_1 = a_2V_2$$

$$V_1 = \frac{Q}{a_1}, \quad V_2 = \frac{Q}{a_2}$$

$$\frac{Q^2}{a_2^2} - \frac{Q^2}{a_1^2} = 2g(h - h_L)$$

$$\Rightarrow Q^2\left[\frac{a_1^2 - a_2^2}{a_1^2a_2^2}\right] = 2g(h - h_L)$$

$$\Rightarrow Q^2 = \frac{a_1^2a_2^2 \cdot 2g(h - h_L)}{a_1^2 - a_2^2}$$

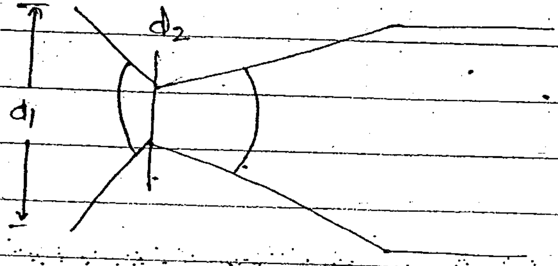
$$\Rightarrow Q_{act} = \frac{a_1a_2\sqrt{2g(h - h_L)}}{\sqrt{a_1^2 - a_2^2}}$$

Comparing with $Q_{act} = \frac{C_d\, a_1a_2\sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$

$$\Rightarrow \frac{\cancel{a_1a_2} \times \sqrt{2g(h - h_L)}}{\cancel{\sqrt{a_1^2 - a_2^2}}} = \frac{C_d\cancel{a_1a_2}\sqrt{2gh}}{\cancel{\sqrt{a_1^2 - a_2^2}}}$$

$$\Rightarrow C_d = \sqrt{\frac{h - h_L}{h}}$$

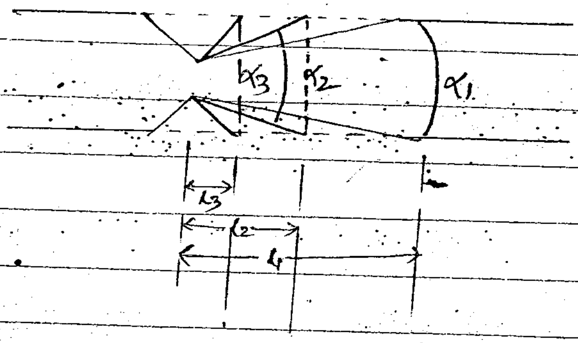
General proportions of a Venturimeter



$$d_2 = \left(\frac{1}{3} \text{ to } \frac{1}{2}\right) d_1$$

Angle of Convergence = 21–22°

Angle of divergence = 5 to 7°



The diverging angle is never kept more than 7° to avoid flow separation

Q 72 A Venturimeter of 20 mm throat diameter is used to measure discharge in a horizontal pipe of 40mm diameter. The fluid flowing is Water. The pressure difference b/w throat s/c and pipe is 30kpa then find the velocity of pipe.

Ans. $d_1 = 40\,mm$

$d_2 = 20mm$

$P_1 - P_2 = 30 \times 10^3\ KN/m^2$

$w = 9810\,N/m^2$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

$$\Rightarrow \frac{P_1 - P_2}{w} = \frac{V_2^2 - V_1^2}{2g}$$

$$\Rightarrow \frac{P_1 - P_2}{e\cancel{g}} = \frac{V_2^2 - V_1^2}{2\cancel{g}}$$

$$\Rightarrow \frac{30 \times 10^3}{10^3} = \frac{V_2^2 - V_1^2}{2}$$

$$\Rightarrow 30 = \frac{1}{2}\left[V_2^2 - V_1^2\right]$$

$$a_1 V_1 = a_2 V_2$$

$$\Rightarrow \frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2$$

$$\Rightarrow 40^2 V_1 = 20^2 V_2$$

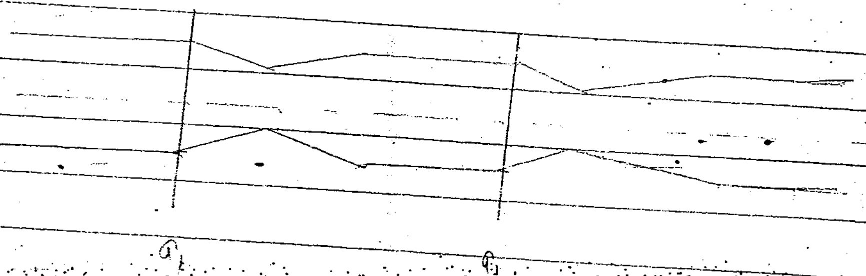
$$\Rightarrow 4V_1 = V_2$$

$$\Rightarrow \cancel{160}\ 60 = \left(16V_1^2 - V_1^2\right)$$

$$\Rightarrow V_1 = 2 m/s$$

Q73. 2 venturimeter of different area ratios are Connected at different location of same pipeline, Similar manometers are used across two ventarimeters to measure discharge. The first Venturimeter of area ratio 2, registers a pressure head of 'h' While the 2nd Venturimeter registers pressure head '~~h~~ 5h' Then find the area ratio of 2nd Venturimeter.

Ans. As both are Connected in same pipe line ∴ so a_1 is same, the difference is that throat areas are different.



$$Q = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$\Rightarrow Q = \frac{a_1 a_2 \sqrt{2gh}}{a_2 \sqrt{\frac{a_1^2}{a_2^2} - 1}} \quad \Rightarrow Q = \frac{a_1 \sqrt{2gh}}{\sqrt{a_r^2 - 1}}$$

$$\Rightarrow \frac{a_1 \sqrt{2gh}}{\sqrt{2^2 - 1}} = \frac{a_1 \sqrt{2g5h}}{\sqrt{a_{r_2}^2 - 1}}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{5}}{\sqrt{a_{r_2}^2 - 1}}$$

$$\Rightarrow a_{r_2}^2 - 1 = 15 \quad \Rightarrow a_{r_2} = 4$$

**

Q14 A Venturimeter is installed in a pipe line of 400mm diameter. The throat diameter is $\frac{1}{3}$ of pipe diameter. The pressure in the pipeline is 1.405 kgf/cm² and Vaccum in throat is 37.5cm of mercury. If 4% of differential head is lost b/w gauges then find the discharge in pipe, The fluid flowing is water.

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Ans.

$d_1 = 0.4m$

$a_1 = \frac{\pi}{4}(0.4)^2$

$a_1 = 0.1256 m^2$

$d_2 = \frac{1}{3}d_1 = \frac{1}{3}(0.4)$

$a_2 = \frac{\pi}{4}\left(\frac{0.4}{3}\right)^2$

$\Rightarrow a_2 = 0.0139 m^2$

$h_L = \frac{4}{100}h$

$\Rightarrow h_L = 0.04h$

$Cd = \sqrt{\frac{h-h_L}{h}} = \sqrt{\frac{h-0.04h}{h}}$

$\Rightarrow Cd = 0.979$

$h = \frac{P_1 - P_2}{w}$

$\Rightarrow P_1 = 1.405 \, kgf/cm^2$

$\Rightarrow 1kgf = 9.81N$

$\Rightarrow P_1 = \frac{1.405 \times 9.81)}{(10^{-2})^2 m^2}$

$\Rightarrow P_1 = 137830.5 N/m^2$

$P_2 = 37.6$ cm of Hg (Vacoom)

$$P_2 = -13.6 \times 10^3 \times 9.81 \times \frac{37.5}{100}$$

$$\Rightarrow P_2 = -50031 N/m^2$$

$$h = \frac{P_1 - P_2}{w} = \frac{137830.5 - (-50031)}{9810}$$

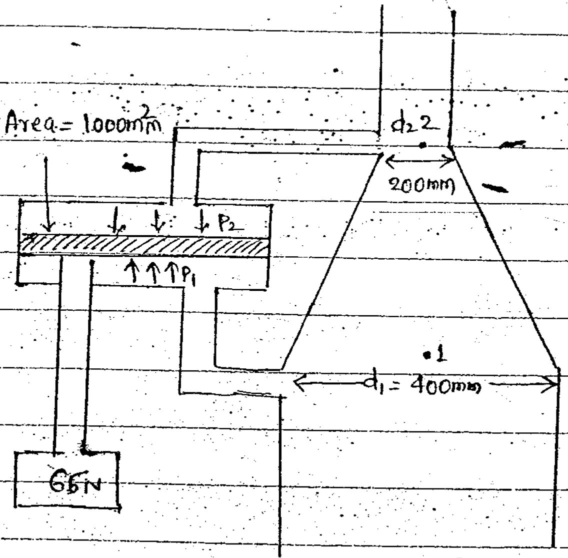
$$\Rightarrow h = 19.15 m$$

$$\Rightarrow Q = \frac{C_d a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}} = 0.2654 m^3/sec.$$

Q75 Oil of specific gravity 0.8 flows upwards through a Vertical Venturimeter as shown in fig. find the discharge through the Venturimeter neglecting frictional losses, X-s/cnal area of piston rod and also neglect potential energy changes

Ans.

$$Q = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$



$$a_1 = \frac{\pi}{4}(0.4)^2, \quad h = \frac{P_1 - P_2}{w}$$

$$a_2 = \frac{\pi}{4}(0.2)^2, \quad w = \rho g$$

$$= 800 \times 9.81$$

$$w = 7848$$

$$\Rightarrow P_2 A + 65 = P_1 A$$

$$\Rightarrow (P_1 - P_2) A = 65$$

$$\Rightarrow (P_1 - P_2) = \frac{65}{1000 \times (10^{-3})^2}$$

$$\Rightarrow P_1 - P_2 = 65 \times 10^3 N/m^2.$$

$$Q = \frac{a_1 a_2 \sqrt{2gh}}{\sqrt{a_1^2 - a_2^2}}$$

$$h = \frac{P_1 - P_2}{w}$$

$$\Rightarrow w = \rho g = 800 \times 9.81$$

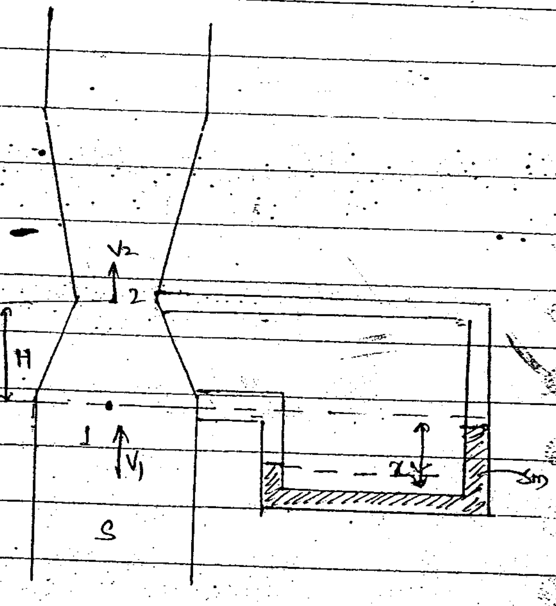
$$w = 7848$$

$\therefore$

$$h = \frac{65 \times 10^3}{7848}$$

$$\Rightarrow Q = 0.413 \text{ m}^3/\text{sec}$$

Q76 The pressure difference across a Venturimeter as shown in fig. is measured with the help of mercury manometer. To estimate the discharge of water, then find the expression for Velocity of Water at the throat.



Ans. $$\frac{P_1}{w} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + Z_2$$

$$\Rightarrow \frac{P_1}{w} + \frac{V_1^2}{2g} + 0 = \frac{P_2}{w} + \frac{V_2^2}{2g} + H$$

$$\frac{P_1}{w} - \frac{P_2}{w} - H = \frac{V_2^2 - V_1^2}{2g} \quad —— ①$$

$$\frac{P_1}{w} + x - \frac{x S_m}{S} - H = \frac{P_2}{w}$$

$$\frac{P_1}{w} - \frac{P_2}{w} - H = x\left(\frac{S_m}{S} - 1\right) \quad —— ②$$

from ① & ②

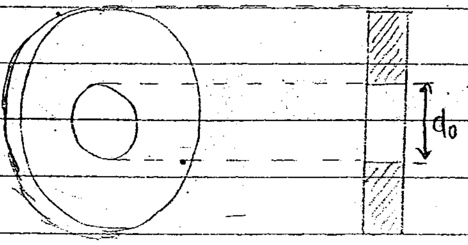
$$\frac{V_2^2 - V_1^2}{2g} = x\left(\frac{S_m}{S} - 1\right)$$

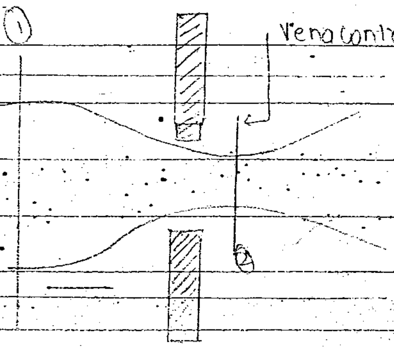
*** Discharge remains same irrespective of positioning of

Venturimeter (i.e. inclined, horizontal or Vertical) -

5/1/2012

Orifice Meter :- It is used for finding out discharge.





Venacontracta (it occurs far from orifice due to inertia of Contraction)

$$C_c = \frac{a_2}{a_0} \quad \text{(Coefficient of Contraction)}$$

$$a_2 = C_c a_0$$

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g} \Rightarrow \frac{P_1 - P_2}{w} = \frac{V_2^2 - V_1^2}{2g} = h$$

$$V_2^2 - V_1^2 = 2gh$$

$$a_1 V_1 = a_2 V_2 \Rightarrow V_1 = \frac{a_2 V_2}{a_1} \Rightarrow V_1 = \frac{C_c a_0 V_2}{a_1}$$

$$\Rightarrow V_2^2 - \frac{C_c^2 a_0^2 V_2^2}{a_1^2} = 2gh$$

$$\Rightarrow V_2^2\left[1-\frac{C_c^2 a_0^2}{a_1^2}\right] = 2gh$$

$$\Rightarrow V_2 = \frac{\sqrt{2gh}}{\sqrt{1-\frac{C_c^2 a_0^2}{a_1^2}}}$$

$$Q = a_2 V_2 = \frac{C_c a_0 \sqrt{2gh}}{\sqrt{1-\frac{C_c^2 a_0^2}{a_1^2}}}$$

$$\Rightarrow Q = \frac{C_c a_0 \sqrt{2gh}}{\sqrt{1-\frac{C_c^2 a_0^2}{a_1^2}}} \times \frac{\sqrt{1-\frac{a_0^2}{a_1^2}}}{\sqrt{1-\frac{a_0^2}{a_1^2}}}$$

$$\Rightarrow Q = \frac{C_d\, a_0 \sqrt{2gh}}{\sqrt{1-\frac{a_0^2}{a_1^2}}}$$

$$\Rightarrow \boxed{Q = \frac{C_d\, a_1 a_0 \sqrt{2gh}}{\sqrt{a_1^2-a_0^2}}}$$

As the area reduction is sudden in orifice meter losses are more and hence Cd of orifice meter is in the range of 0.682 to 0.74.

$$C_d = \frac{Q_{act}}{Q_{the}} = \frac{A_{act}}{A_{th}} \frac{V_{act}}{V_{th}}$$

← C_v (Coeff. of Velocity)

↑ Coefficient of Contraction

$$\boxed{C_d = C_c \times C_v}$$

Q77. A orificemeter with Cd = 0.61 is substituted by a venturimeter with Cd = 0.98 in a pipe line carrying oil having same throat diameter as that of orifice meter. For the same flow rate the ratio of pressure drops for ~~same~~ Venturimeter to that of orificemeter is

(a). $\frac{0.61}{0.98}$ (b) $\frac{0.98}{0.61}$ (c) $\frac{0.61^2}{0.98^2}$ (d) $\frac{0.98^2}{0.61^2}$

Ans. Q_v (discharge of Venturimeter) = Q_o (disch. of orificemeter).

for Venturimeter

$$Q_v = \frac{C_{dv}\, a_1 a_2 \sqrt{2gh_v}}{\sqrt{a_1^2 - a_2^2}}$$

for orifice meter

$$Q_o = \frac{C_{do}\, a_1 a_o \sqrt{2gh_o}}{\sqrt{a_1^2 - a_o^2}}$$

as $d_2 = d_o$

$\Rightarrow a_2 = a_o$ (i.e. Throat area of Venturimeter = orifice area of orifice meter).

$$\Rightarrow \frac{C_{dv} \cancel{a_1} \cancel{a_2} \sqrt{2gh_v}}{\cancel{\sqrt{a_1^2 - a_2^2}}} = \frac{C_{do} \cancel{a_1} \cancel{a_o} \sqrt{2gh_o}}{\cancel{\sqrt{a_1^2 - a_o^2}}}$$

$$\Rightarrow C_{dv}\sqrt{h_v} = C_{do}\sqrt{h_o}$$

$h = \frac{P_1 - P_2}{w} = \frac{\Delta P}{w}$

$$\Rightarrow C_{dv}\sqrt{\Delta P_v} = C_{do}\sqrt{\Delta P_o}$$

$\Rightarrow h \propto \Delta P$

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$$\frac{\sqrt{\Delta P_v}}{\sqrt{\Delta P_0}} = \frac{Cd_0}{Cd_v}$$

$$\Rightarrow \frac{\Delta P_v}{\Delta P_0} = \frac{Cd_0^2}{Cd_v^2} = \frac{0.61^2}{0.98^2}$$

Q78 An orifice meter is being used for measuring discharge of a liquid in a pipeline shows a pressure head differential of xm of water column when the discharge is Q. If the discharge is doubled then the pressure head differential in 'm' of water colum is

(a) $2x$ (b) $4x$ (c) $x/2$ (d) $3x$

Ans. $$Q = \frac{Cd\, a_1 a_0 \sqrt{2gh}}{\sqrt{a_1^2 - a_0^2}}$$

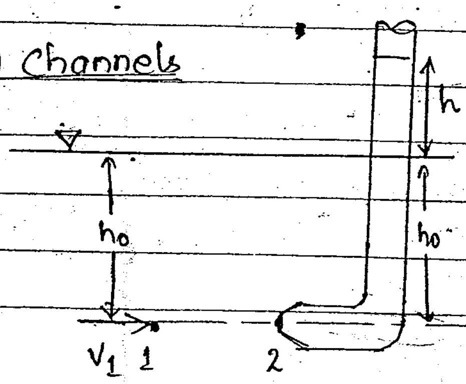
$$\Rightarrow Q \propto \sqrt{h}$$

$$\Rightarrow \frac{Q_2}{Q_1} = \frac{\sqrt{h_2}}{\sqrt{h_1}} \Rightarrow \frac{2Q}{Q} = \sqrt{\frac{h_2}{h}}$$

$$\Rightarrow 4 = \frac{h_2}{x} \Rightarrow \underline{\underline{h_2 = 4x}}$$

Pitot Tube :- It is used to find out the velocity of flow

Case - I Velocity in open channels



$P_1 = \rho g h_0$

$\Rightarrow h_0 = \frac{P_1}{\rho g} \Rightarrow h_0 = \frac{P_1}{w}$

$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \cancel{\frac{V_2^2}{2g}} \rightarrow 0$ (as fluid is static at orifice)

$\Rightarrow \frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w}$

$\rho g h + \rho g h_0 = P_2$

$\Rightarrow \rho g (h + h_0) = P_2$

$\Rightarrow (h + h_0) = \frac{P_2}{\rho g} = \frac{P_2}{w}$

$\Rightarrow \frac{P_1}{w}$ is h_0, i.e. static head

$\Rightarrow \cancel{h_0} + \frac{V_1^2}{2g} = h + \cancel{h_0}$

$\Rightarrow \frac{V_1^2}{2g} = h$

$\Rightarrow \frac{V_1^2}{2g} = h =$ Dynamic head

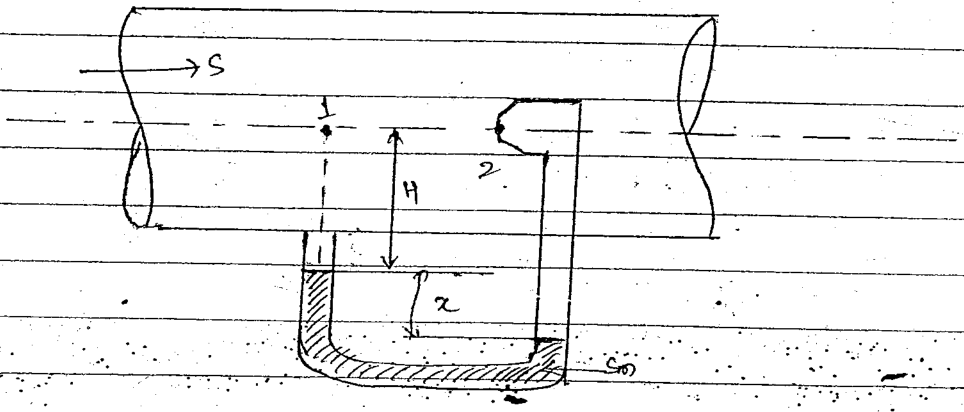
$$\underset{\substack{\uparrow \\ \text{static head}}}{\frac{P_1}{w}} + \underset{\substack{\uparrow \\ \text{dynamic head}}}{\frac{V_1^2}{2g}} = \underset{\substack{\uparrow \\ \text{stagnation head}}}{\frac{P_2}{w}}$$

Static head + Dynamic head = Stagnation head.

Dynamic head = Stagnation head - Static head

$$\Rightarrow V_1 = \sqrt{2g\left(\underset{\text{head}}{\text{stagnation}} - \underset{\text{head}}{\text{static}}\right)}$$

Case-II Velocity in pipes



$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w}$$

$$\Rightarrow \frac{V_1^2}{2g} = \frac{P_2}{w} - \frac{P_1}{w} \quad \text{——} ①$$

$$x \times S_m = h \times S$$

$$\Rightarrow h = \frac{x \times S_m}{S}$$

$$\frac{P_1}{W} + \cancel{H} + \frac{x S_m}{S} - x - \cancel{H} = \frac{P_2}{w}$$

$$\Rightarrow \frac{P_2 - P_1}{w} = x\left(\frac{S_m}{s} - 1\right) \quad \text{——} ②$$

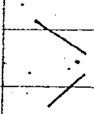
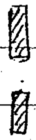
from ① and ②

$$\frac{V_1^2}{2g} = x\left(\frac{S_m}{S}-1\right)$$

$$\Rightarrow \quad V_1 = \sqrt{2gx\left(\frac{S_m}{S}-1\right)}$$

$x \rightarrow$ Manometric fluid deflection

Stagnation state is the state when the fluid is brought to rest isentropically.

Device	Shape	Cost	losses	Cd
Venturimeter		more	less	more
flow nozzle or Nozzle meter		medium	medium	medium
Orifice meter		less	more	less

Q79. A pipot tube is used to measure velocity of water using a differential gauge which contains manometric fluid of specific gravity 1.4. If the velocity of water in pipe is 1.2 m/s then find the manometric fluid deflection.

Ans. $V = 1.2\ m/s$

$S_m = 1.4$, $S = 1$ (Water)

$$V = \sqrt{2gx\left(\frac{S_m}{S} - 1\right)}$$

$$\Rightarrow 1.2 = \sqrt{2 \times 9.81 \times x \times \left(\frac{1.4}{1} - 1\right)}$$

$$\Rightarrow x = \underline{\underline{0.183\ m}}$$

LAMINAR FLOW

Reynold's Number :- It is the ratio of Inertia force to the viscous force

$$Re = \frac{\rho v L}{\mu} = \frac{VL}{(\mu/\rho)} = \frac{VL}{\nu}$$

For pipes

L → Characteristic length
In pipes → L = D

$Re < 2000$ → laminar
$Re > 4000$ → Turbulent

$2000 < Re < 4000$ → Transition

Darcy - weisbach Eqn

This eqn is used for finding out head loss due to friction in pipes. This eqn is applicable for both laminar and turbulent flow but the flow should be steady.

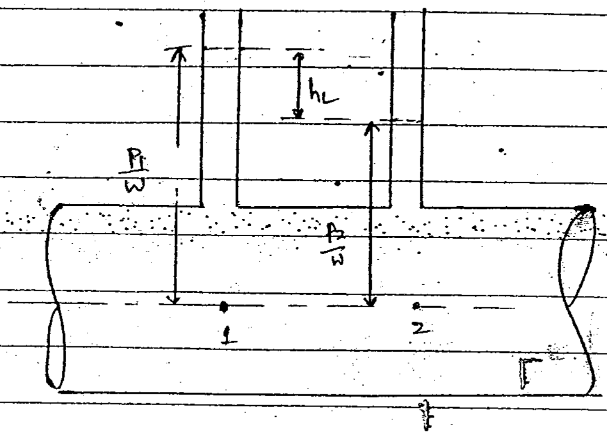
$$h_L = \frac{4f'LV^2}{2gD}, \quad f' \rightarrow \text{friction coeff}$$

$$h_L = \frac{fLV^2}{2gD}, \quad f \rightarrow \text{friction factor}$$

$f = 4f'$

The above eqns can be applied in a ~~flow~~ pipe of Constant x-s/c and pipe may be horizontal, vertical or inclined.

Bernoulli's Cannot be used for Laminar flow (viscous flow) as it is applied for ideal flow (non-viscous flow).
So We use modified Bernoulli's eqn.



$$\frac{P_1}{w} + \cancel{\frac{V_1^2}{2g}} + z_1 = \frac{P_2}{w} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_L \quad \left(\begin{matrix} A_1V_1 = A_2V_2 \\ \Rightarrow V_1 = V_2 \end{matrix} \right)$$

$$\Rightarrow \left(\frac{P_1}{w} + Z \right) = \left(\frac{P_2}{w} + Z_2 \right) + h_L$$

$$\Rightarrow \left(\frac{P_1}{w} + z_1 \right) - \left(\frac{P_2}{w} + Z_2 \right) = h_L$$

Horizontal $\Rightarrow z_1 = z_2$

$$\Rightarrow \boxed{\frac{P_1}{w} - \frac{P_2}{w} = h_L} \quad (I)$$

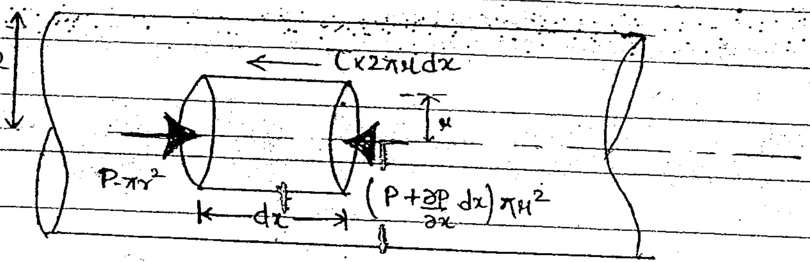
$$\Rightarrow \frac{P_1}{w} = h_L + \frac{P_2}{w}$$

Pressure decreases in dirn of flow in order to overcome losses. (i.e. Pressure gradient is -ve in the dirn of flow)

Laminar Flow Through Circular Pipes (Hagen - Poiseulle flow)

Assumptions

- Steady flow
- fully developed flow (Velocity profile doesnot Vary in longitudinal dirn)



as after fully developed flow there is no change in Velocity
∴ acceleration = 0 (i.e. Convective + Local acc = 0).

$$P\pi r^2 - \left(P + \frac{\partial P}{\partial x}dx\right)\pi r^2 - 2\tau\pi r\, dx = 0$$

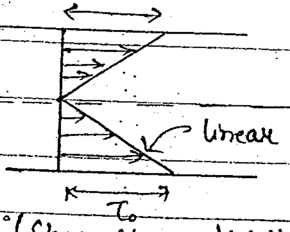
$$\Rightarrow Pr - \left(P + \frac{\partial P}{\partial x}dx\right)r - 2\tau dx = 0$$

$$\Rightarrow \cancel{Pr} - \cancel{Pr} - \frac{\partial P}{\partial x}dx \times r - 2\tau dx = 0$$

$$\Rightarrow -\frac{\partial P}{\partial x}dx \cdot r = 2\tau dx$$

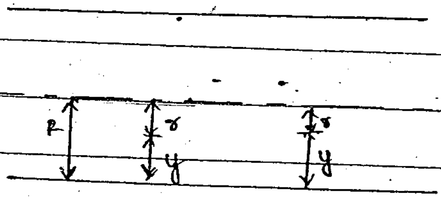
$$\Rightarrow -\frac{\partial P}{\partial x} \cdot r = 2\tau$$

$$\Rightarrow \boxed{\tau = -\frac{\partial P}{\partial x} \cdot \frac{r}{2}} \quad ①$$



(Shear Stress distribution)

Velocity Distribution



$R = r + y$

$0 = dr + dy$

$dy = -dr$

$\Rightarrow$ as $\tau = \mu \frac{du}{dy}$ $\Rightarrow \tau = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$

$\Rightarrow \tau = -\frac{\partial p}{\partial x} \cdot \frac{r}{2} = -\mu \frac{du}{dr}$

$\Rightarrow \cancel{-} \frac{\partial p}{\partial x} \cdot \frac{r}{2} = \cancel{-} \mu \frac{du}{dr}$

$\Rightarrow du = \frac{1}{2\mu} \frac{\partial p}{\partial x} \cdot r\, dr$

$\Rightarrow u = \frac{1}{2\mu} \frac{\partial p}{\partial x} \times \frac{r^2}{2} + C$

At the pipe wall, $r = R$, $u = 0$

$\Rightarrow 0 = \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2 + C$

$\Rightarrow C = \frac{-1}{4\mu} \frac{\partial p}{\partial x} \times R^2$

$\Rightarrow u = \frac{1}{4\mu} \frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu} \frac{\partial p}{\partial x} R^2$

711

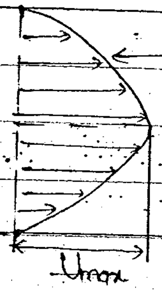
$$\Rightarrow u = -\frac{1}{4\mu}\cdot\left(\frac{\partial p}{\partial x}\right)\left[R^2 - r^2\right]$$

$$u = -\frac{1}{4\mu}\,\frac{\partial p}{\partial x}\,R^2\left[1-\frac{r^2}{R^2}\right]$$

At the centre, i.e. $r = 0 \Rightarrow U = U_{max}$

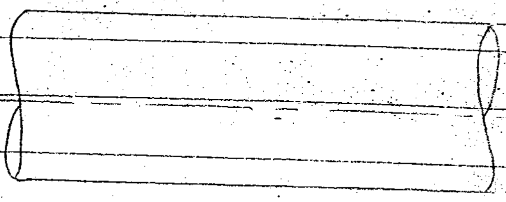
$$\Rightarrow \boxed{U_{\overset{mean}{max}} = -\frac{1}{4\mu}\left(\frac{\partial p}{\partial x}\right)R^2} \text{ —— } ②$$

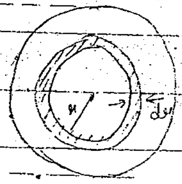
$$\boxed{u = U_{max}\left(1-\frac{r^2}{R^2}\right)} \text{ —— } ③$$



Before fully developed, the flow is called establishing flow.

Discharge (θ)





$$u = U_{max}\left(1-\frac{r^2}{R^2}\right)$$

$$d\theta = U_{max}\left(1-\frac{r^2}{R^2}\right)2\pi r\,dr$$

$$Q = \int_0^R U_{max}\left(1-\frac{r^2}{R^2}\right) 2\pi r\,dr$$

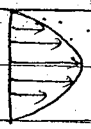
$$Q = 2\pi U_{max}\int_0^R \left(r - \frac{r^3}{R^2}\right)dr$$

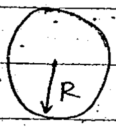
$$\Rightarrow Q = 2\pi \underset{ax}{U_{mean}}\left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R$$

$$\Rightarrow Q = 2\pi U_{max}\left[\frac{R^2}{4}\right]$$

$$\boxed{Q = \frac{\pi U_{max} R^2}{2}} \quad \text{④}$$

Average Velocity

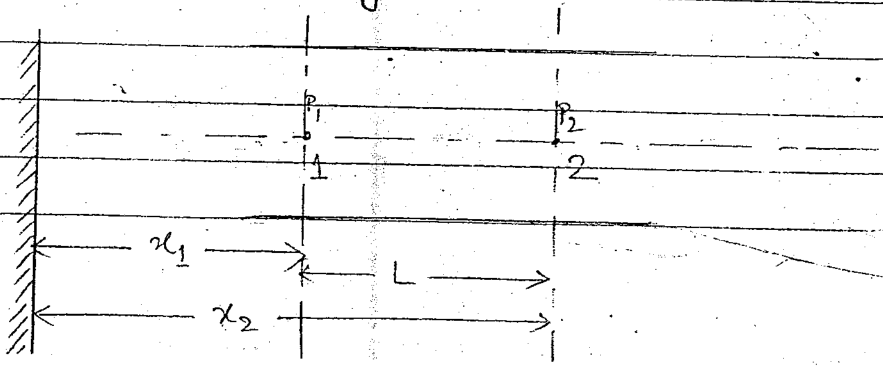




It is the ratio of total discharge to the total area of flow

$$V = \frac{Q}{A} = \frac{\frac{\pi U_{max} R^2}{2}}{\pi R^2} \Rightarrow \boxed{V = \frac{U_{max}}{2}} \quad \text{⑤}$$

Pressure Drop in a length L :-



$$V = \frac{1}{2} U_{max}$$

$$V = \frac{1}{2}\left[\frac{-1}{4\mu}\left(\frac{\partial P}{\partial x}\right)R^2\right]$$

$$\int_{x_1}^{x_2} \frac{8\mu V}{R^2}\,\partial x = \int_{P_1}^{P_2} -\partial P$$

$$\frac{8\mu V}{R^2}[x_2 - x_1] = -[P_2 - P_1]$$

$$\Rightarrow \frac{8\mu V L}{R^2} = P_1 - P_2$$

$$\frac{8\mu V L}{\left(\frac{D}{2}\right)^2} = P_1 - P_2 \Rightarrow \boxed{P_1 - P_2 = \frac{32\mu V L}{D^2}} \text{ —— ⑥}$$

$$\frac{P_1 - P_2}{w} = h_L \Rightarrow P_1 - P_2 = w h_L$$

$$\Rightarrow P_1 - P_2 = \rho g \times \frac{fLV^2}{2gD}$$

$$\Rightarrow \frac{32\mu V L}{D^2} = \frac{\rho f L V^2}{2D}$$

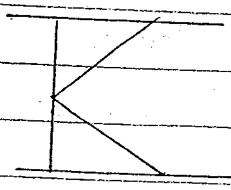
$$\frac{64\mu}{D} = \rho V f$$

$$\Rightarrow f = \frac{64\mu}{\rho V D} \Rightarrow \boxed{f = \frac{64}{Re}} \text{ —— ⑦}$$

In laminar flow friction factor depends only on Reynold's no.

6/1/2012 Shear Velocity

$$\tau = -\frac{\partial P}{\partial x}\frac{r}{2}$$



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At $r = R$, $\tau = \tau_0$

$$\tau_0 = -\frac{\partial p}{\partial x} \cdot \frac{R}{2}$$

$$\Rightarrow \tau_0 = -\frac{(P_2 - P_1)}{(x_2 - x_1)} \cdot \frac{R}{2}$$

$$\Rightarrow \tau_0 = \left(\frac{P_1 - P_2}{L}\right) \times \frac{1}{2} \times \frac{D}{2}$$

$$\Rightarrow \tau_0 = \frac{(P_1 - P_2) D}{4L} \quad \Rightarrow \tau_0 = \left(\frac{\rho f g L V^2}{2gD}\right) \times \frac{D}{4L}$$

$$\Rightarrow \boxed{\tau_0 = \frac{\rho f V^2}{8}}$$

as $h_L = \frac{P_1 - P_2}{\rho g}$

$$\frac{\tau_0}{\rho} = \frac{f}{8} V^2$$

$$\Rightarrow \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{f}{8}}\, V$$

$$\Rightarrow \boxed{\text{Shear velocity } (V_*) = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{f}{8}} \cdot V}$$

Q80. Find the minimum value of friction factor that can occur in laminar flow through circular pipes.

Ans $f = \frac{64}{Ra}$

$f_{min} = \frac{64}{Re_{max}}$

(Re)max for laminar flow is $\frac{64}{\cancel{2000}}$ 2000 ⇒ $f_{min} = 0.032$

Q81. In a laminar flow through a pipe of 10cm, the avg. velocity is 5m/s. then find the velocity at 5cm radius.

Ans. $R = 10cm$, $V = 5m/s$
$r = 5cm$, $u = ?$

$u = U_{max}\left(1 - \frac{r^2}{R^2}\right)$ ⇒

and $V = \frac{U_{max}}{2}$ ⇒ $U_{max} = 10m/s$

⇒ $u = 10\left(1 - \frac{5^2}{10^2}\right) = 7.5m/s$

Q82. In a laminar flow through pipe of diameter 'D', the avg. velocity is indicated as local velocity at a radial distance measured from the centre of ~~at a radial dist~~

Ans $U = U_{max}\left(1 - \frac{r^2}{R^2}\right)$

$V = \frac{U_{max}}{2}$ ⇒ $U_{max} = 2V$

$\cancel{u} = 2\cancel{V}\left(1 - \frac{r^2}{R^2}\right)$ ⇒ $\frac{1}{2} = 1 - \frac{r^2}{R^2}$ ⇒ $\frac{r^2}{R^2} = \frac{1}{2}$

⇒ $\frac{r}{R} = \frac{1}{\sqrt{2}}$ ⇒ $r = 0.707R$ ⇒ $r = 0.707 \times \frac{D}{2}$ $= 0.354D$ Ans.

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Q83. Oil flows from 1 to 2 through a 100m long steel pipe of 150mm diameter, the pressure at 1 is 1.08 Mpa and at 2 is 0.95 MPa. The kinematic viscosity is 412.5×10^{-6} m²/s, $\rho = 918$ kg/m³, then find Reynolds number and discharge if the flow is laminar.

Ans.

$P_1 = 1.08\times10^6 N/m^2$

$P_2 = 0.95\times10^6 N/m^2$

$\Rightarrow \nu = 412.5\times10^{-6} m^2/sec$, $\rho = 918 kg/m^3$

$\nu = \frac{\mu}{\rho} \Rightarrow \mu = \nu\times\rho = 412.5\times10^{-6}\times918 = 0.3786 \frac{N\text{-}s}{m^3}$

$P_1 - P_2 = \frac{32\mu VL}{D^2}$

$\Rightarrow 1.08\times10^6 - 0.95\times10^6 = \frac{32\times0.3786\times V\times100}{(0.15)^2}$

$\Rightarrow V = 2.414$ m/s

$Q = AV = \frac{\pi}{4}\times(0.15)^2\times2.414 = \underline{0.0426 m^3/sec}$

$Re = \frac{\rho VD}{\mu} = \frac{918\times2.417\times0.15}{0.3786} = \underline{877}$

* * *

Q84. Show that the max. shear stress at the pipe wall for laminar flow through a pipe of given diameter 'D' with given fluid properties μ and ρ is $\tau_0 = \frac{16000\mu^2}{\rho D^2}$

Ans.

$\tau_0 = \frac{\rho f V^2}{8}$

$f = \frac{64}{Re} = \frac{64}{\frac{\rho VD}{\mu}} \Rightarrow f = \frac{64\mu}{\rho VD}$

$\Rightarrow \tau_0 = \frac{\rho V^2}{8}\cdot\frac{64\mu}{\rho VD} \Rightarrow \tau_0 = \frac{8\mu V}{D}$

$$\tau_0 = \frac{\overset{\text{fixed}}{8}\mu V}{\underset{\text{fixed}}{D}}$$

τ_0 is max; when 'V' is max.

$$\text{fixed}\ \frac{\rho V \overset{\text{fixed}}{D}}{\underset{\text{fixed}}{\mu}} = Re$$

⇒ 'V' is max when Re is max.

from laminar flow, max $Re = 2000$

$$\frac{\rho V D}{\mu} \Rightarrow = 2000 \Rightarrow V = \frac{2000\mu}{\rho D}$$

$$\Rightarrow \tau_0 = \frac{8\mu}{D} \times \frac{2000\mu}{\rho D}$$

$$\Rightarrow \tau_0 = \frac{16000\mu^2}{\rho D^2}$$

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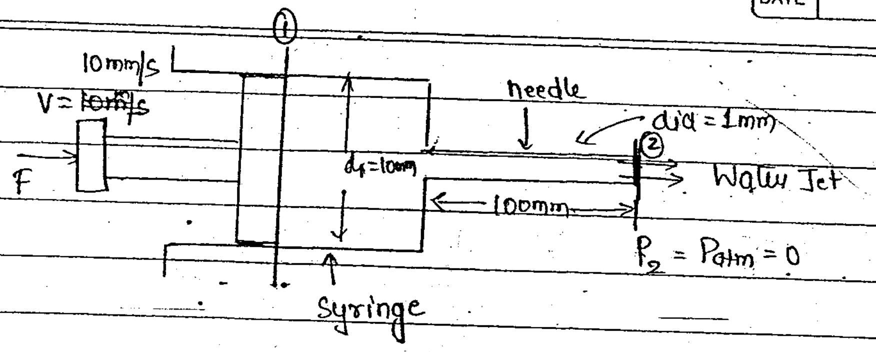
Q85. A Syringe with a frictionless plunger, and has at its end a $\overset{100\,mm}{\wedge}$ long needle of ~~10~~ 1mm diameter. The internal diameter of the Syringe is 10mm, Water density is 1000 kg/m³. The plunger is pushed in at 10mm/sec and water comes out ~~as~~ as jet to atmosphere.

(a). Assuming Ideal flow, find the force 'F' reqd on the plunger to push out water.

(b). Neglect losses in the Syringe and assume laminar flow through out the needle find the force F. Take viscosity of Water as $10^{-3}\,\frac{Ns}{m^2}$.



Ans (a). $V_1 = 10\times10^{-3}$ m/sec.

$a_1V_1 = a_2V_2$

$$\Rightarrow \frac{\pi}{4}\times(10)^2\times10\times10^{-3} = \frac{\pi}{4}\times1^2\times V_2$$

$$\Rightarrow V_2 = 1\,m/s$$

$$\frac{P_1}{w}+\frac{V_1^2}{2g} = \cancel{\frac{P_2}{w}}^{0}+\frac{V_2^2}{2g}$$

$$\Rightarrow \frac{P_1}{\rho g} = \frac{V_2^2-V_1^2}{2g}$$ { $h_L = 0$, no losses, ideal fluid }

$$\Rightarrow P_1 = \frac{\rho}{2}(V_2^2-V_1^2)$$

$$\Rightarrow P_1 = \frac{10^3}{2}\left[1^2-(10\times10^{-3})^2\right]$$

$$\Rightarrow P_1 = 499.95\,N/m^2$$

$$\Rightarrow F = P_1A_1 \quad \Rightarrow F = 499.95\times\frac{\pi}{4}(10\times10^{-3})^2 = 0.039N$$

(b) There is no losses in Syringe, but it is given flow is laminar in needle ⇒ that there is viscosity then there will be head losses.

$$\frac{P_1}{w}+\frac{V_1^2}{2g} = \frac{P_2}{w}+\frac{V_2^2}{2g}+h_L$$

$$\Rightarrow \frac{P_1}{w} = \frac{V_2^2-V_1^2}{2g}+h_L$$

$$h_L = \frac{fLv^2}{2gD}$$

$L = 100mm = 0.1m$, $D = 1mm \times 10^{-3}m$, $V = 1m/s$

$\Rightarrow f = \frac{64}{Re}$, $Re = \frac{\rho VD}{\mu} = \frac{10^3 \times 1 \times 10^{-3}}{10^{-3}} = 1000$

$\Rightarrow f = \frac{64}{1000}$

$\Rightarrow h_L = \frac{0.064 \times 0.1 \times 1^2}{2 \times 9.81 \times 10^{-3}}$ $\Rightarrow h_L = 0.326m$

$\Rightarrow \frac{P_1}{9810} = \frac{1^2 - [10 \times 10^{-3}]^2}{2 \times 9.81} + 0.326$

$\Rightarrow P_1 = 3698 \ N/m^2$

$\Rightarrow F = P_1A_1 = 3698 \times \frac{\pi}{4}(10 \times 10^{-3})^2$

$= \underline{\underline{0.29N}}$

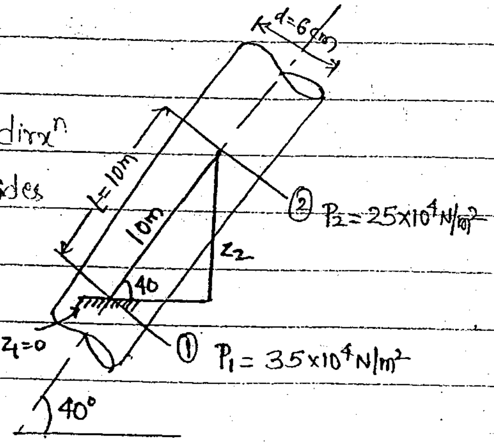
Q86. An oil of density 900 kg/m³ and kinematic viscosity 0.0002 m²/s flows upwards through an inclined pipe as shown in fig. The pressures at the two sections '1' and '2' which are 10m apart are $P_1 = 35 \times 10^4$ N/m² and $P_2 = 25 \times 10^4$ N/m². Assuming steady laminar flow, verify that the flow is upwards and find :

(a). Head loss b/w '1' and '2'.

(b). Velocity

(c). Discharge

(d). Reynold's number.



Ans. It is not only pressure that decides the dirxⁿ of flow. It is the total energy that decides the dirxⁿ of flow.

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(a). Piezometric head $P_{H_1} = \frac{P_1}{w} + z_1 = \frac{35\times10^4}{9.8\times900} + 0$

$\Rightarrow P_{H1} = 39.64m$

$PH_2 = \frac{P_2}{w} + z_2 = \frac{25\times10^4}{900\times9.81} + 10\sin40°$

$\Rightarrow PH_2 = 34.74\ m$

as $PH_1 > PH_2 \Rightarrow$ flow is from ① to ②

As it is laminar flow,

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2 + h_L$$

$A_1V_1 = A_2V_2 \Rightarrow V_1 = V_2$ as $A_1 = A_2$

$\Rightarrow \frac{P_1}{w} + z_1 = \left(\frac{P_2}{w} + z_2\right) + h_L$

$\Rightarrow PH_1 = PH_2 + h_L$

$\Rightarrow 39.64 = 34.74 + h_L$

$\Rightarrow h_L = 4.9m$

Since fluid is climbing up, pressure is also reducing by its weight according to hydrostatic

$l\rho g = 900\times9.81\times10\sin40° = P_h$

$\Rightarrow V_{max} = \frac{-1}{4\mu}\frac{(P_1 - P_2 - P_h)}{L}\times(0.03)^2$

$\Rightarrow V_{mean} = \frac{V_{max}}{2} = \underline{\underline{2.7m/s}}$

as increasing P_2 by making it horizontal.

(b). $h_L = \frac{fLV^2}{2gD}$, $f = \frac{64}{Re} = \frac{64\mu}{\rho VD}$

$\Rightarrow h_L = \frac{64\mu}{\rho VD}\times\frac{LV^2}{2gD} = \frac{32\mu}{\rho}\times\frac{LV}{gD^2} = \frac{32\nu VL}{gD^2}$

$\Rightarrow 4.9 = \frac{32\times0.0002\times V\times10}{9.81\times(6\times10^{-2})^2}$

$\Rightarrow V = 2.7\ m/s$

$Q = AV = \frac{\pi}{4}(6\times10^{-2})^2\times2.7$

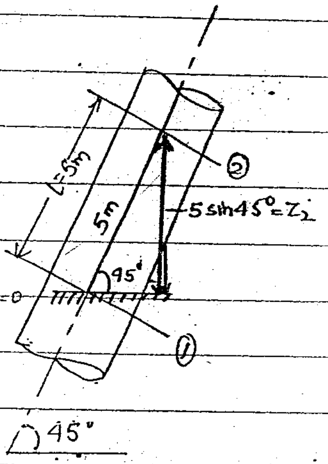
$\Rightarrow Q = 7.62\times10^{-3}\ m^3/sec.$

and $Re = \frac{\rho VD}{\mu} = \frac{VD}{\mu/\rho} \Rightarrow Re = \frac{VD}{\nu} = \frac{2.7\times6\times10^{-2}}{0.0002} = \underline{\underline{810}}$

087. An upward flow of oil of density 800kg/m³ and viscosity 0.8 Ns/m² takes place under laminar cond^ns in an inclined pipe of 0.1m diameter as shown in fig. The pressures at section ① and ② are 435 KN/m² and 200 KN/m² resp. find

(a). Discharge in pipe.

(b). If the flow dirn is reversed keeping the same discharge and pressure at section ① is maintained as 435 KN/m². find the pressure at section ②.



Ans. (a) $V_1 = V_2$ (as $A_1 = A_2$)

$$\Rightarrow \frac{P_1}{w} + Z_1 = \frac{P_2}{w} + Z_2 + h_L$$

$$\Rightarrow \frac{435\times10^3}{800\times9.81} + 0 = \frac{200\times10^3}{800\times9.81} + 5\sin45° + h_L$$

$$\Rightarrow h_L = 26.4m$$

$$f = \frac{64}{Re} = \frac{64}{\frac{\rho VD}{\mu}} = \frac{64\mu}{\rho VD}$$

$$h_L = \frac{fLV^2}{2gD} \Rightarrow h_L = \frac{64\mu}{\rho VD}\times\frac{LV^2}{2gD} = \frac{32\mu VL}{\rho gD^2}$$

$$\Rightarrow 26.4 = \frac{32\times0.8\times V\times5}{800\times9.81\times0.1^2} \Rightarrow V = 16.2 m/s$$

$$Q = A\times V = \frac{\pi}{4}\times(0.1)^2\times16.2 \Rightarrow Q = 0.127 m^3/s.$$

(b). $$\frac{P_2}{w} + \cancel{\frac{V_2^2}{2g}} + Z_2 = \frac{P_1}{w} + \cancel{\frac{V_1^2}{2g}} + Z_1 + h_L$$

$$\Rightarrow \frac{P_2}{w} + Z_2 = \frac{P_1}{w} + Z_1 + h_L$$

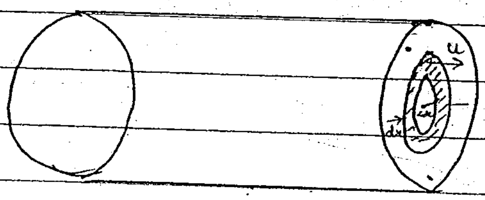
$$\Rightarrow \frac{P_2}{800\times9.81} + 5\sin45^\circ = \frac{435\times10^3}{800\times9.81} + 0 + 26.4$$

$$\Rightarrow P_2 = 614.4\times10^3 \, N/m^2 \text{ Ans.}$$

Momentum Correction Factor (β)

In Continuity eqn and Bernoulli's eqn We use average velocities.

Momentum Correction factor $\beta = \frac{P_{act}}{P_{av}}$



$\Rightarrow P_{avg} = \text{mass}\times\text{vel.}$

$\Rightarrow P_{av} = \rho AV\times V$

$\Rightarrow P_{av} = \rho AV^2$

$$P_{act} = \int(\rho Au)\times u = \int \rho u^2 dA$$

$$\Rightarrow \beta = \frac{\int \rho u^2 dA}{\rho AV^2} = \frac{\int u^2 dA}{AV^2}$$

It is the ratio of momentum based on actual velocity to the momentum based on average velocity.

Kinetic Energy Correction Factor (α)

It is the ratio of K.E based on actual velocity to the K.E based on average velocity.

$$K.E = \frac{1}{2}(mv).v = P\times\frac{V}{2}$$

$P_{av} = \rho AV^2 \rightarrow KE_{av} = \rho AV^2\times\frac{V}{2}$

$P_{act} = \int \rho dA u^2 \rightarrow KE_{act} = \int \rho u^2 dA\times\frac{u}{2}$

$$\Rightarrow \alpha = \frac{(KE)_{act}}{(KE)_{avg.}} = \frac{\int \rho u^3 dA/2}{\rho/2\, AV^3} = \frac{\int u^3 dA}{AV^3} = \alpha$$

Q88. Find the momentum Correction factor for Laminar flow through Circular pipes.

Ans. $A = \pi R^2$; $V = \frac{U_{max}}{2}$

$$\beta = \frac{\int u^2 dA}{AV^2}$$

$$V = \frac{U_{max}}{2}$$

$$u = U_{max}\left(1 - \frac{r^2}{R^2}\right)$$

$$\Rightarrow u^2 = U_{max}^2\left(1 + \frac{r^4}{R^4} - \frac{2r^2}{R^2}\right)$$

$$\Rightarrow \int u^2 dA = \int_0^R U_{max}^2\left[1 + \frac{r^4}{R^4} - \frac{2r^2}{R^2}\right] . 2\pi r\, dr$$

$$\int U^2 dA = 2\pi U_{max}^2 \int_0^R \left(r + \frac{r^5}{R^4} - \frac{2r^3}{R^2}\right) dr$$

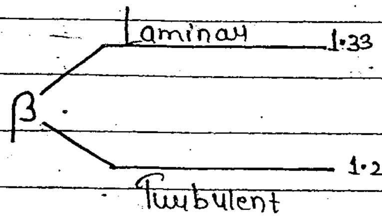
$$\Rightarrow \int U^2 dA = \left[\frac{r^2}{2} + \frac{r^6}{6R^4} - \frac{2r^4}{4R^2}\right]_0^R 2\pi U_{max}^2$$

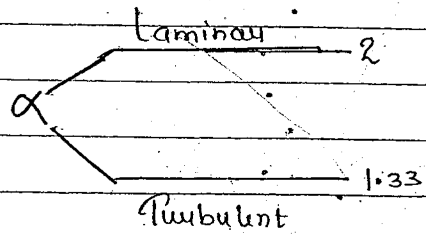
$$\Rightarrow \int U^2 dA = 2\pi U_{max}^2 \left[\frac{R^2}{6}\right]$$

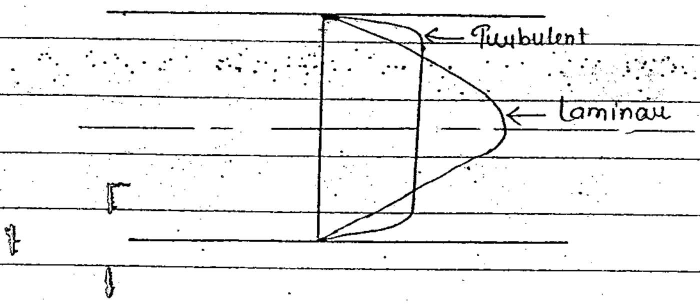
$$\Rightarrow \int U^2 dA = \frac{\pi U_{max}^2 R^2}{3}$$

$$\Rightarrow \beta = \frac{\int U^2 dA}{AV^2} = \frac{\pi U_{max}^2 R}{3} \Big/ \pi R^2 \times \left(\frac{U_{max}}{2}\right)^2$$

$$= 4/3 = 1.33$$







The larger values of correction factor for laminar flow indicates that the deviation in velocity is more compared to turbulent flow. In turbulent flow correction factors are less i.e. the velocity distribution is more uniform due to rapid mixing of fluid particles.

Vortex Motion

The motion of fluid in a curved path is known as Vortex motion. Vortex motion is of two types

- Forced Vortex
- Free Vortex

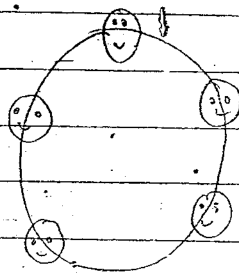
Bernoulli's theorem मे कोई external energy नही देते हैं fluid खुद के energy से flow होता है। Forced Vortex मे external energy देते है so, forced Vortex मे Bernoulli's use नही करते हैं

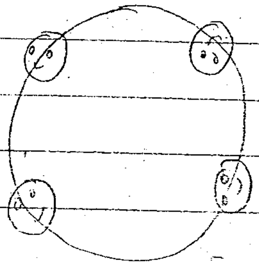
Compressor मे Impeller मे forced Vortex and Casing or diffuser मे free Vortex motion.

In forced Vortex, the fluid moves in curved path under the influence of external torque. As there is continuous expenditure of energy in forced Vortex Bernoulli's eqⁿ is not applicable. For forced vortex, $V = r \times \omega$ is applicable.

Eg:- Liquid in a Container When rotated, Motion of fluid in impeller of a Centrifugal pump.

In free Vortex fluid moves in curved path due to motion given to it earlier. As there is no expenditure of energy Bernoulli's eqⁿ is applicable for free vortex. free vortex is an irrotational flow


Irrotational


Rotational

Eg:- flow of liquid in pipe bend, flow of fluid in diffuser of Centrifugal pump, Whirlpool, flow of liquid in wash basin.

Rate of change of Angular momentum = Torque

$$\Rightarrow \frac{d}{dt}(mv r) = T$$

as T = 0 in free Vortex

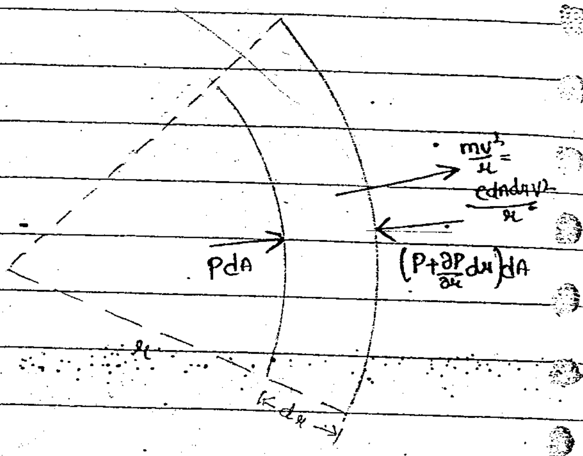
$$\Rightarrow mvr = \text{Constant}$$

$$\Rightarrow vr = \text{Constant}/m$$

$$\Rightarrow \boxed{vr = \text{Constant}}$$

General Equation for Vortex Motion

$$PdA + \frac{\rho\, dA\, dr\, V^2}{r} - \left(P + \frac{\partial P}{\partial r} dr\right) dA = 0$$



$$\Rightarrow \cancel{P} + \frac{\rho V^2}{r} dr = \cancel{P} + \frac{\partial P}{\partial r} dr$$

$$\Rightarrow \frac{\rho V^2}{r} \cancel{dr} = \frac{\partial P}{\partial r} \cancel{dr}$$

$$\Rightarrow \boxed{\frac{\partial P}{\partial r} = \frac{\rho V^2}{r}}$$

The above eqⁿ shows Variation of pressure in radial dirxⁿ

Also pressure changes in vertical dirxⁿ, by hydrostatic law,

$$\frac{\partial P}{\partial z} = \rho g \quad \text{and} \quad \frac{\partial P}{\partial z} = -\rho g \text{ (in upward dirxⁿ)}$$

$$P = f(r, z)$$

$$\Rightarrow dP = \frac{\partial P}{\partial r} dr + \frac{\partial P}{\partial z} dz$$

$$\Rightarrow \boxed{dp = \frac{\rho v^2 dr}{r} - \rho g dz}$$

for both free & forced vortex motion.

Free Vertex Motion

for free vortex motion $V \times r = \text{Constant}$

$$dp = \frac{\rho v^2 dr}{r} - \rho g dz$$

$$\Rightarrow V \times r = C \Rightarrow V = \frac{C}{r}$$

$$\Rightarrow dp = \frac{\rho}{r} \cdot \frac{C^2}{r^2} dr - \rho g dz$$

$$\Rightarrow dp = \rho C^2 \times \frac{dr}{r^3} - \rho g dz$$

$$\Rightarrow \int_{P_1}^{P_2} dp = \int_{r_1}^{r_2} \frac{\rho c^2 dr}{r^3} - \int_{z_1}^{z_2} \rho g dz$$

$$\Rightarrow (P_2 - P_1) = \rho c^2 \left[\frac{-1}{2r^2}\right]_{r_1}^{r_2} - \rho g (z)_{z_1}^{z_2}$$

$$\Rightarrow (P_2 - P_1) = -\frac{\rho c^2}{2} \left[\frac{1}{r_2^2} - \frac{1}{r_1^2}\right] - \rho g (z_2 - z_1)$$

$$\Rightarrow (P_2 - P_1) = -\frac{\rho}{2}\frac{c^2}{r_2^2} + \frac{\rho}{2}\frac{c^2}{r_1^2} - \rho g z_2 + \rho g z_1$$

$$\Rightarrow (P_2 - P_1) = -\frac{\rho V_2^2}{2} + \frac{\rho V_1^2}{2} - \rho g z_2 + \rho g z_1$$

$$\Updownarrow$$

$$P_2 + \frac{\rho V_2^2}{2} + \rho g z_2 = P_1 + \frac{\rho V_1^2}{2} + \rho g z_1$$

Divide with ρg.

$$\Rightarrow \boxed{\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1} \leftarrow \text{Bernoulli's eq}^n$$

Thus above eqn indicates that Bernoulli's eqn is valid in free Vortex motion.

7/1/2012 Forced Vortex Motion

$$dP = \frac{\rho v^2}{r} dr - \rho g dz$$

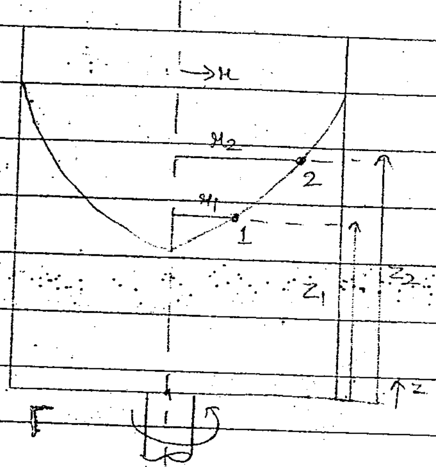
for forced Vortex, $V = r\omega$

$$dP = \frac{\rho}{r} r^2 \omega^2 dr - \rho g dz$$

$$\int_{P_1}^{P_2} dp = \int_{r_1}^{r_2} \rho \omega r^2 dr - \int_{z_1}^{z_2} \rho g dz$$

$$\Rightarrow P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - g(z_2 - z_1)\rho$$

Let us select points ① and ② on the surface $P_1 = P_2 = P_{atm}$



$$\Rightarrow 0 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$$

$$\Rightarrow \frac{\cancel{\rho}\omega^2}{2}(r_2^2 - r_1^2) = \cancel{\rho}g(z_2 - z_1)$$

$$\Rightarrow \frac{\omega^2}{2}(r_2^2 - r_1^2) = g(z_2 - z_1)$$

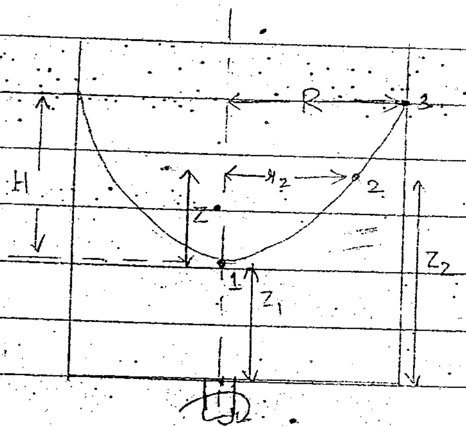
let us take point 1 on the axis of rotation, $\Rightarrow r_1 = 0$

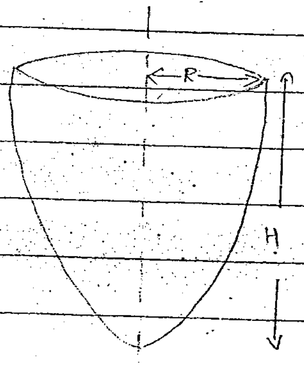
$$\Rightarrow \frac{\omega^2 r_2^2}{2} = g(z_2 - z_1)$$

$$\Rightarrow \frac{\omega^2 r_2^2}{2} = gz \quad \text{where } z_2 - z_1 = z$$

for point at 3;

$$\frac{\omega^2 R^2}{2g} = H$$





Volume of Paraboloid = $\frac{1}{2}\pi R^2 H$

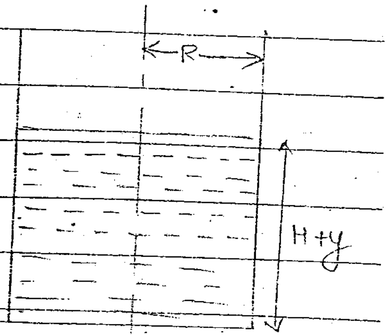
Q89. Show that in case of forced vortex, the rise of liquid levels at the ends is equal to fall of liquid at axis of rotation when no water spills out.

Ans. Initial Volume = $\pi R^2(H+y)$

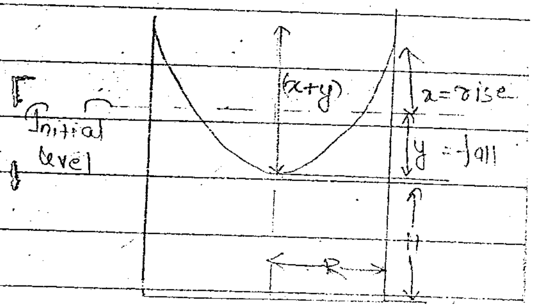
$= \pi R^2(H+x+y) - \frac{1}{2}\pi R^2(x+y)$

$\Rightarrow \cancel{H}+\cancel{y} = \cancel{H}+x+\cancel{y} - \frac{1}{2}(x+y)$

$0 = x - \frac{x}{2} - \frac{y}{2} \Rightarrow x = y$



This is the condn when no water spills out.

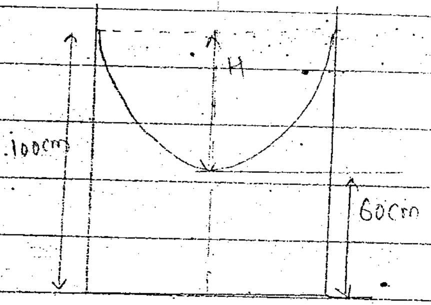


Q90 A open cylinder of 20cm dia. and 100 cm long contains water upto a height of 60cm. The tank is rotated about vertical axis at 300 r.p.m. Find the depth of paraboloid formed.

Ans. $\frac{\omega^2 R^2}{2g} = H$

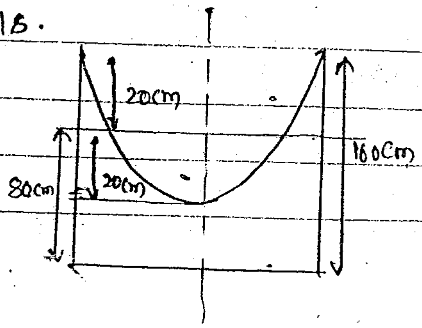
$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 31.4$ rad/s

$\Rightarrow H = \frac{31.4^2 \times (0.1)^2}{2\times 9.81} = 0.502m$



Q91. An open cylinder of 15 cm diameter and 100cm long contains water upto a height of 80cm. Find the maximum speed at which the cylinder is rotated so that no water spills.

Ans. As no water spills out then rise of water will equal to fall in water level



$\Rightarrow H = \frac{W^2R^2}{2g} \Rightarrow \frac{W^2(7.5\times10^{-2})^2}{2\times9.81} = 0.4$

$\Rightarrow W = 37.3$ rad/sec

$\Rightarrow W = \frac{2\pi N}{60} \Rightarrow N = 356.67$ rpm.

Q92 A Cylindrical vessel 12cm diameter & 30cm height is filled with Water up to the top. The vessel is open at the top. Find the quantity of water left in the Container when it is rotated about its axis at a speed of

(a). 300 rpm

(b). 600 rpm.

Ans. (a). $W = \frac{2\pi N}{60} = 31.42$ rad/s

⇒ Height of paraboloid that will be formed

$\Rightarrow H = \frac{W^2R^2}{2g} = 18.1$ cm

⇒ Volume of Paraboloid = $\frac{1}{2}\pi R^2 \times H$ = ~~325.86 cm³~~ 1023.53 cm³.

and Volume of Water initially = $\pi \times 6^2 \times 30$ = 3393 cm³

⇒ Volume of Water left = ~~3067.12 cm³~~ 2369.47 cm³ Ans.

(b). $W = \frac{2\pi 600}{60} = 62.84$ rad/s

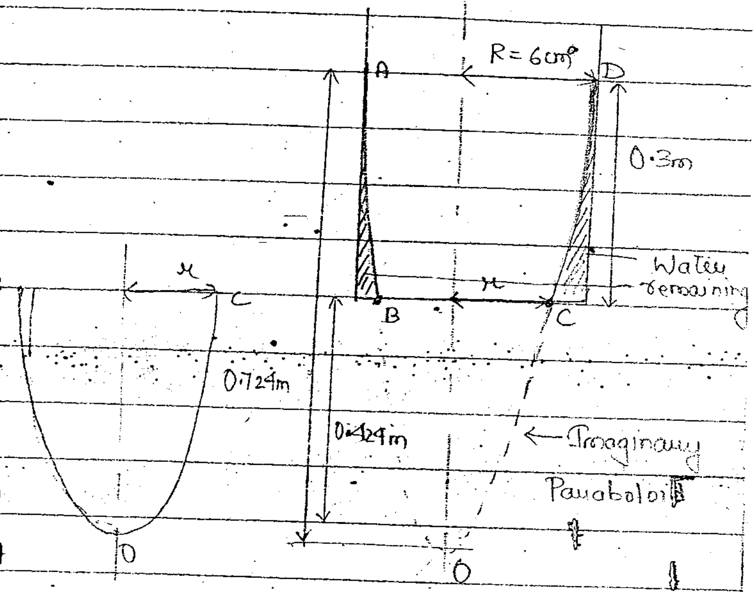
⇒ Height of paraboloid that will be formed

$\Rightarrow H = \frac{(62.84)^2 \times (0.06)^2}{2\times9.81} = 72.4$ cm.

⇒ Volume of paraboloid = $\frac{1}{2}\pi R^2 H = \frac{1}{2} \times \pi \times (6)^2 \times 72.4$ = 4094.12 cm³ now, $\frac{\omega^2 r^2}{2g} = h$

$\Rightarrow \frac{62.4^2 \times r^2}{2 \times 9.81} = 0.424$

$\Rightarrow r = 0.0459m$



Volume of AOD (larger paraboloid)

$= \frac{1}{2}\pi R^2 H = 4.09 \times 10^{-3} m^3$

Volume of BOC (smaller paraboloid)

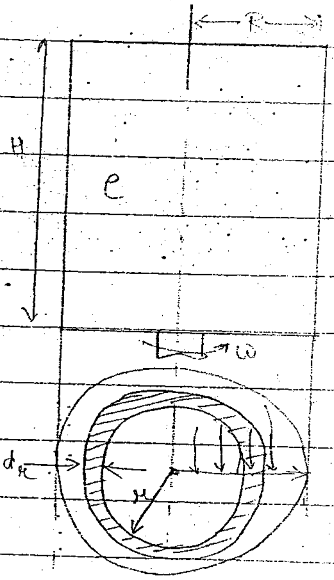
$= \frac{1}{2}\pi r^2 h = \frac{1}{2}\pi (0.0459)^2 \times 0.424$

$= 1.403 \times 10^{-3} m^3$

$\Rightarrow$ Vol. ABCD = $4.09 \times 10^{-3} - 1.403 \times 10^{-3} = 2.69 \times 10^{-3} m^3$

$\Rightarrow$ Vol. left = $3.39 \times 10^{-3} - 2.69 \times 10^{-3} = 0.69 \times 10^{-3} m^3$ Ans.

Q93. A closed cylinder having radius 'R' and height 'H' is filled with Oil of density 'e'. If the cylinder is rotated about its vertical axis at a constant angular velocity 'ω', then find the force at the bottom of the cylinder.



Ans. If the container had been stationary then at the bottom, weight of liquid had been acting. But, as it is rotated there will be radial variation of pressure along the radius of bottom of cylinder.

Since pressure is varying, so taking an small circular element and analysing

Wt = egv

$\Rightarrow f_1 = Wt = \rho g \pi R^2 H$

$$\frac{\partial p}{\partial r} = \frac{\rho v^2}{r}$$ ✓

$V = \omega r$ ✓

$$\Rightarrow \frac{\partial p}{\partial r} = \frac{\rho \omega^2 r^2}{r} \Rightarrow \frac{\partial p}{\partial r} = \rho \omega^2 r$$ ✓

$$\Rightarrow \partial p = \rho \omega^2 r \, \partial r \Rightarrow p = \frac{\rho \omega^2 r^2}{2}$$ ✓

$$dF = PdA, \Rightarrow dF = \frac{\rho \omega^2 r^2}{2} \times 2\pi r dr$$ ✓

$$\Rightarrow dF = \pi \rho \omega^2 r^3 dr$$

$$f_2 = \int_0^R \pi \rho \omega^2 r^3 dr$$

$$\Rightarrow f_2 = \pi \rho \omega^2 \left[\frac{r^4}{4} \right]_0^R$$

$$\Rightarrow f_2 = \frac{\pi \rho \omega^2}{4} \times R^4$$

$$\Rightarrow \text{Total force } f = f_1 + f_2 = \rho g \pi R^2 H + \frac{\pi}{4} \rho \omega^2 R^4$$

** The force on the top of the cylinder is only due to pressure but not due to weight ∴ the force on the top of cylinder = f_2 ∴ the critical portion is the bottom one from where the container when rotated has the chance of failure.

FLOW THROUGH THE PIPES

When fluid flows through pipes it encounters various losses and these losses are classified into

- Major losses
- Minor losses

Major losses are head loss due to friction, losses due to sudden expansion, sudden contraction, bend loss are known as minor losses.

Major Losses

(a). Darcy - Weisbach eqⁿ (It is applicable for both laminar and turbulent flow)

$$h_L = \frac{fLV^2}{2gD}, \quad Q = AV$$

$$\Rightarrow Q = \frac{\pi D^2 V}{4}$$

$$\Rightarrow V = \frac{4Q}{\pi D^2}$$

$$\Rightarrow h_L = \frac{fL}{2gD} \cdot \frac{16Q^2}{\pi^2 D^4} = \frac{fL Q^2}{\frac{2g\pi^2}{16} D^5} = \frac{fLQ^2}{12 D^5}$$

Imp. When discharge is given & for use in parallel and series connection of pipes.

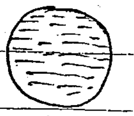
Generally the length of pipes are large so $h_L = \frac{fLV^2}{2gD}$ is large
∴ h_L is a major loss and accounts for 90-95% loss.

(b) Chezy's formula

$$V = C\sqrt{mi}$$

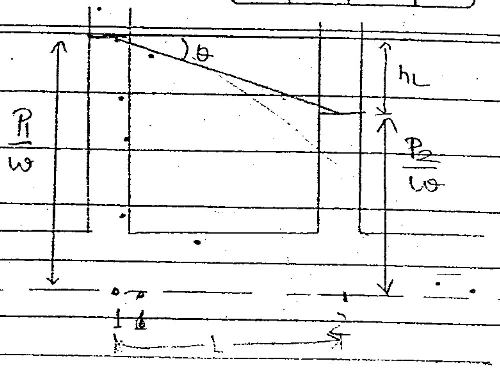
C → Chezy's Constant

m → hydraulic mean depth (Wetted Perimeter)



$$m = \frac{A}{P} = \frac{\pi D^2}{4 \pi D} = \frac{D}{4}$$

$i =$ hydraulic Slope



$i =$ Slope $= \tan\theta = \frac{h_L}{L}$

$\Rightarrow i = \frac{h_L}{L}$

$$\Rightarrow \quad V = C\sqrt{\frac{D}{4} \times \frac{h_L}{L}}$$

$$\Rightarrow \quad V^2 = c^2 \frac{D h_L}{4L}$$

$$\Rightarrow \quad \boxed{h_L = \frac{4LV^2}{c^2 D}}$$

now Comparing with Darcy- Weisbach eqn,

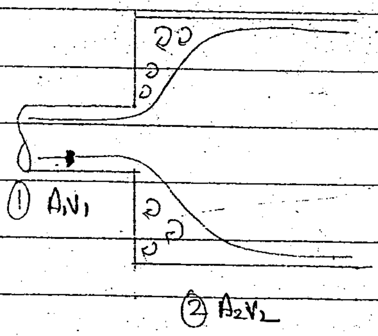
$$\Rightarrow \quad \frac{fLV^2}{2gD} = \frac{4LV^2}{c^2 D}$$

$$\Rightarrow \quad \frac{f}{2g} = \frac{4}{c^2} \Rightarrow c^2 = \frac{8g}{f} \Rightarrow \boxed{C = \sqrt{\frac{8g}{f}}}$$

← Relation b/w Chezy's Const. and friction factor.

Minor Losses

0 Sudden Expansion Loss

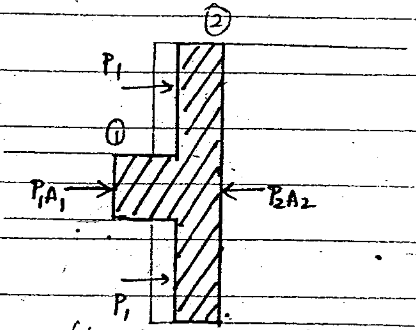


By momentum Eq^n ——— (a)

$\Sigma F = ma$

$\Rightarrow \Sigma f = m\left(\frac{v-u}{t}\right)$

$\Rightarrow \Sigma F = \overset{o}{m}(v-u) \Rightarrow \Sigma F = \rho Q(v-u)$



(found experimentally equal and Logically length small so Pressure drop less)

now,

$P_1A_1 + P_1(A_2 - A_1) - P_2A_2 = \rho Q(V_2 - V_1)$

$\Rightarrow P_1A_2 - P_2A_2 = \rho Q(V_2 - V_1)$

$\Rightarrow P_1 - P_2 = \frac{\rho Q}{A_2}(V_2 - V_1)$

$\Rightarrow$ by Continuity eq^n ——— (b)

$Q = A_1V_1 = A_2V_2$

$\Rightarrow A_2 = \frac{Q}{V_2} \Rightarrow V_2 = \frac{Q}{A_2}$

$\Rightarrow P_1 - P_2 = \rho V_2(V_2 - V_1)$

$\Rightarrow \frac{P_1 - P_2}{\rho} = V_2(V_2 - V_1)$

Now, by Bernoulli's eq^n, ——— (c)

$$\frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g} + h_{Lexp}$$

$$\Rightarrow \frac{P_1 - P_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} = h_{Lexp}$$

$$\Rightarrow \frac{V_2(V_2 - V_1)}{g} + \frac{V_1^2 - V_2^2}{2g} = h_{Lexp}$$

$$\Rightarrow h_{Lexp} = \frac{V_1^2 + V_2^2 - 2V_1V_2}{2g} = \frac{(V_1 - V_2)^2}{2g}$$

$$\Rightarrow h_{Lexp} = \frac{V_1^2}{2g}\left[1 - \frac{V_2}{V_1}\right]^2$$

$A_1V_1 = A_2V_2$

$$\Rightarrow \frac{V_2}{V_1} = \frac{A_1}{A_2}$$

$$\Rightarrow \boxed{h_{L\,exp} = \frac{V_1^2}{2g}\left[1 - \frac{A_1}{A_2}\right]^2}$$

3 principles are employed in deriving this expression.

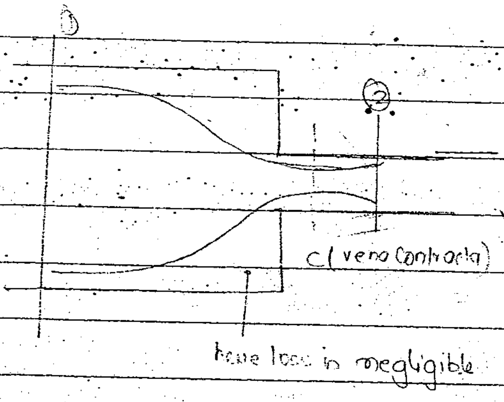
o **Loss at Exit**

It is similar to sudden expansion with $A_2 = \infty$,

$$\therefore \boxed{\text{Exit loss} = \frac{V^2}{2g}}$$

o **Sudden Contraction loss.**

Vena-Contracta is always in Sudden Contraction, Converging area में losses कम होते हैं पर diverging मे ज्यादा होता है।



$$a_c V_c = a_2 V_2$$

$$\Rightarrow \frac{V_c}{V_2} = \frac{a_2}{C_c} \Rightarrow \frac{V_c}{V_2} = \frac{1}{C_c}$$

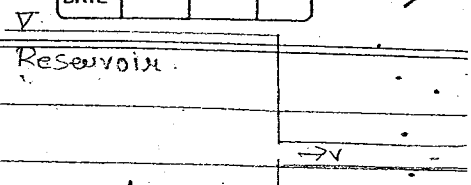
$C_c \rightarrow$ Coeff. of Contraction,

$$h_{L\,Contraction} = \frac{(V_c - V_2)^2}{2g}$$

$$= \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2$$

$$\boxed{h_{L\,Contraction} = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2}$$

* * If C_c is not given, then take Sudden Contraction losses as $\frac{0.5 V_2^2}{2g}$ Where V_2 is Velocity in Smaller diameter pipe.



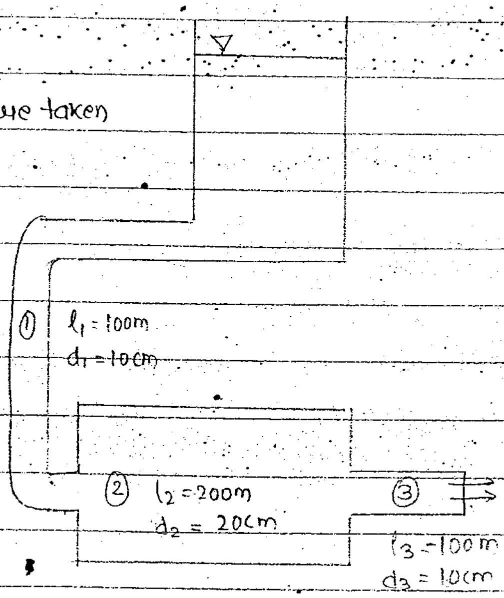
- Entrance Loss

It is similar to Sudden Contraction and hence entrance loss is equal to $\frac{0.5V^2}{2g}$, Where V is Velocity in ~~smaller~~ pipe (as there is only one pipe).

- Bend Loss

Bend loss = $\frac{KV^2}{2g}$, Where K is a Constant Which depends on radius of Curvature and angle of bend.

Q94. Water flows from a reservoir through a series of pipes Joined as shown in fig. find the percentage error in discharge if minor losses are neglected. Assume K=1 for bends and friction factor $f = 0.\overset{0}{_\wedge}2$ for all pipes, and the available head of 20m is used in overcoming losses.



Ans Case I :- Discharge When all losses are taken into account.

Equating discharges,

$$\Rightarrow a_1V_1 = a_2V_2 = a_3V_3$$

$$\Rightarrow \frac{\pi}{4}d_1^2 \times V_1 = \frac{\pi}{4}d_2^2 \times V_2 = \frac{\pi}{4}d_3^2 \times V_3$$

$$\Rightarrow \frac{\cancel{\pi}}{\cancel{4}} \times (10)^2 \times V_1 = \frac{\cancel{\pi}}{\cancel{4}}(20)^2 \times V_2 = \frac{\cancel{\pi}}{\cancel{4}} \times (10)^2 \times V_3$$

$$\Rightarrow V_1 = 4V_2 = V_3$$

Total available head = 20m,

$$\Rightarrow 20 = \frac{0.5V_1^2}{2g} + \frac{KV_1^2}{2g} + \frac{f_1L_1V_1^2}{2gd_1} + \frac{KV_1^2}{2g} + \frac{(V_1-V_2)^2}{2g} + \frac{f_2L_2V_2^2}{2gd_2} + \frac{0.5V_3^2}{2g} + \frac{f_3l_3V_3^2}{2gd_3} + \frac{V_3^2}{2g}$$

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Substituting V_2 and V_3 in terms of V_1

$\Rightarrow$ $V_2 = \frac{V_1}{4}$ and $V_3 = V_1$

$$\Rightarrow \frac{0.5V_1^2}{2g} + \frac{V_1^2}{2g} + \frac{0.02 \times 100 \times V_1^2}{2g \times 0.1} + \frac{1 \times V_1^2}{2g} + \frac{(V_1 - V_1/4)^2}{2g} + \frac{0.02 \times 200 \times V_1^2}{16 \times 2g \times 0.2}$$

$$+ \frac{0.5V_1^2}{2g} + \cancel{1\times}\, \frac{0.02 \times 100 \times V_1^2}{2g \times 0.1} + \frac{V_1^2}{2g} = 20$$

$\Rightarrow$ $V_1 = 2.93$ m/s

$\Rightarrow$ $Q_{act} = A_1V_1 = \frac{\pi}{4} d_1^2 \times V_1 = \frac{\pi}{4} (0.1)^2 \times 2.93 = 0.023$ m³/sec.

Case II :- neglecting minor losses

$$\Rightarrow 20 = \frac{f_1 L_1 V_1^2}{2gd_1} + \frac{f_2 L_2 V_2^2}{2gd_2} + \frac{f_3 L_3 V_3^2}{2gd_3}$$

$\Rightarrow$ Substituting V_2 and V_3 in terms of V_1

$$\Rightarrow \frac{0.02 \times 100 \times V_1^2}{2g \times 0.1} + \frac{0.02 \times 200 \times V_1^2}{16 \times 2g \times 0.2} + \frac{0.02 \times 100 \times V_1^2}{2g \times 0.1} = 20$$

$\Rightarrow$ $V_1 = 3.08$ m/s

$\Rightarrow$ $Q = A_1V_1 = \frac{\pi}{4} (0.1)^2 \times 3.08 = 0.0242$ m³/sec

Error is found with respect to actual.

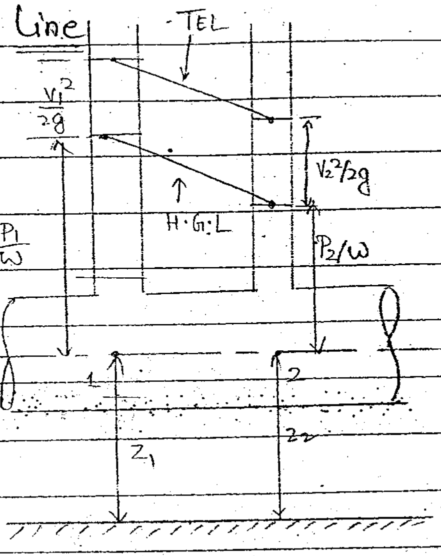
$\Rightarrow$ percentage error $= \frac{0.0242 - 0.023}{0.023} \times 100 = \underline{\underline{5.2\%}}$

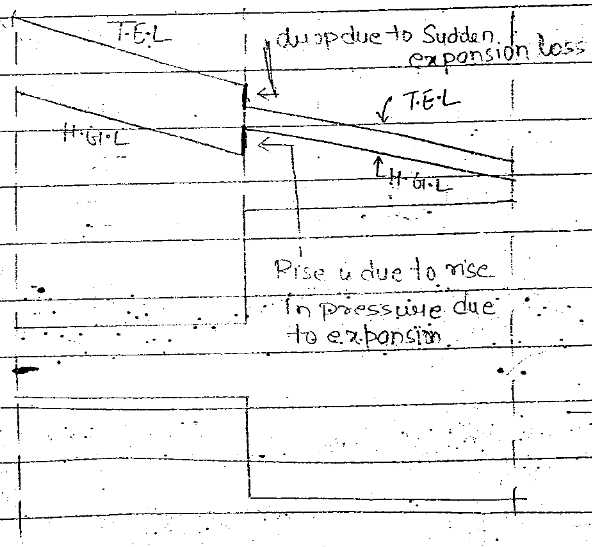
Hydraulic gradient line and Total Energy Line

The line which joins piezometric heads at various points in a flow is known as hydraulic gradient line.

The line which joins the total head at various points in a flow is known as Total energy line.

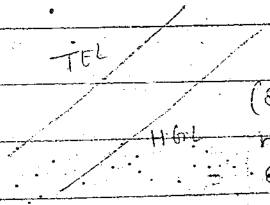
The distance b/w HGL and TEL gives kinetic energy head or velocity head.





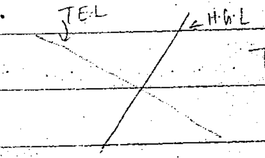
There may be types.

(I)



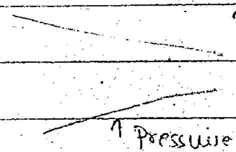
(Energy increases with supply of Energy)

(II)



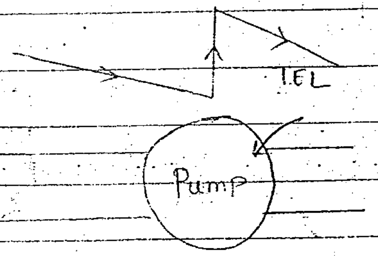
T.E.L can never be less than H.E.L

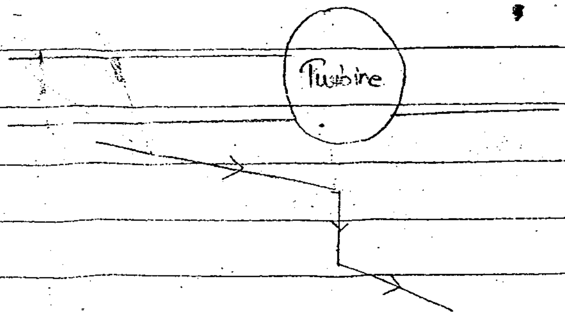
(III)



It is also wrong.

Pressure decreasing not increasing.





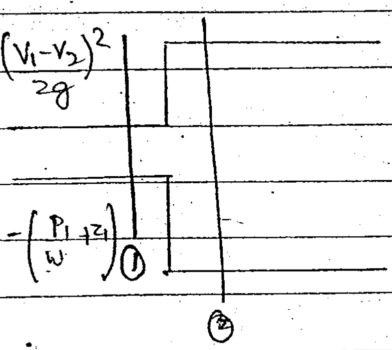
The hydraulic gradient line can rise or fall but the total energy line will rise only when there is external energy input as in case of pump.

9/01/2012

Q95. At a sudden expansion of water pipe line from a diameter from 0.24m to 0.48m, the hydraulic gradient line rises by 10mm. then find the discharge.

Ans. Rise in Hydraulic gradient line represents piezometric head.

$$\frac{P_1}{w}+Z_1+\frac{V_1^2}{2g}=\frac{P_2}{w}+Z_2+\frac{V_2^2}{2g}+\frac{(V_1-V_2)^2}{2g}$$



$$\Rightarrow \frac{V_1^2}{2g}-\frac{V_2^2}{2g}-\frac{(V_1-V_2)^2}{2g}=\left(\frac{P_2}{w}+Z_2\right)-\left(\frac{P_1}{w}+Z_1\right)$$

$$\Rightarrow \frac{V_1^2-V_2^2-\left[V_1^2+V_2^2-2V_1V_2\right]}{2g}=\frac{10}{1000}$$

$$\Rightarrow \frac{2V_1V_2-2V_2^2}{2g}=0.01$$

$$\Rightarrow \frac{V_1V_2-V_2^2}{g}=0.01$$

$$A_1V_1=A_2V_2 \Rightarrow \frac{\pi}{4}\times(0.24)^2\times V_1=\frac{\pi}{4}\times(0.48)^2\times V_2$$

$$\Rightarrow V_1=4V_2$$

$$\Rightarrow \frac{4V_2.V_2-V_2^2}{g}=0.01$$

$$\Rightarrow \frac{3V_2^2}{g}=0.01 \quad \Rightarrow V_2=0.18\,m/s$$

$$\Rightarrow Q=A_2V_2=\frac{\pi}{4}(0.48)^2\times 0.18=0.0327\,m^3/sec.$$

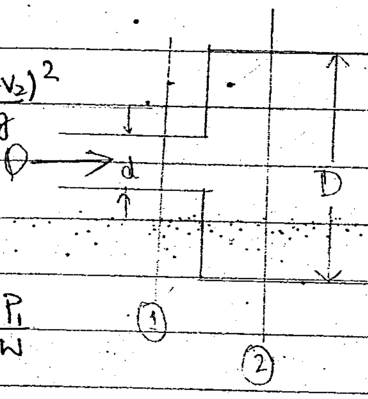
***

Q96. A horizontal pipe of given diameter 'd' suddenly enlarges to 'D', then find the ratio d/D, such that the rise in pressure for a given discharge past the enlargement shall be maximum..

Ans.

$$\frac{P_1}{w} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{w} + \frac{V_2^2}{2g} + \cancel{Z_2} + \frac{(V_1-V_2)^2}{2g}$$

(as horizontal)



$$\Rightarrow \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - \frac{(V_1-V_2)^2}{2g} = \frac{P_2}{w} - \frac{P_1}{w}$$

$$\Rightarrow \frac{\cancel{V_1^2} - V_2^2 - (\cancel{V_1^2} + V_2^2 - 2V_1V_2)}{2g} = \frac{P_2 - P_1}{w}$$

$$\Rightarrow \frac{2V_1V_2 - V_2^2}{2g} = \frac{\Delta P}{\rho g}$$

$$\Rightarrow V_1V_2 - V_2^2 = \frac{\Delta P}{\rho}$$

$$\Rightarrow \Delta P = \rho(V_1V_2 - V_2^2)$$

for max pressure rise, $\frac{d(\Delta P)}{dV_2} = 0$

$$\Rightarrow \frac{d(\Delta P)}{dV_2} = \rho\left[V_1(1) - 2V_2\right] = 0$$

$$\Rightarrow V_1 = 2V_2$$

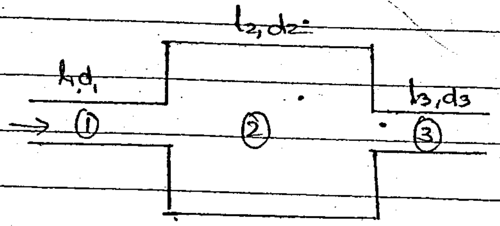
$$Q = A_1V_1 = A_2V_2 \quad \Rightarrow \frac{\pi}{4}d_1^2V_1 = \frac{\pi}{4}d_2^2V_2$$

$$\Rightarrow d^2V_1 = D^2V_2$$

$$\Rightarrow \frac{D^2}{d^2} = \frac{V_1}{V_2} \quad \Rightarrow \frac{D^2}{d^2} = 2$$

$$\Rightarrow \frac{D}{d} = \sqrt{2}$$

Pipes In Series

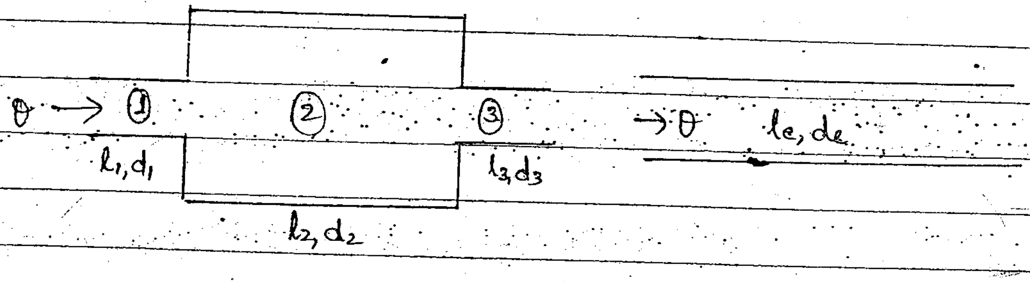


$$\theta_1 = \theta_2 = \theta_3 = \theta$$

and $h_L = h_{L_1} + h_{L_2} + h_{L_3}$ (neglecting minor losses).

Equivalent Pipe

A pipe of uniform diameter is said to be equivalent to a compound pipe if it carries same discharge and also encounters same losses.



$$h_{Le} = \frac{f L_e \theta^2}{12 d_e^5}$$

$$\Rightarrow \quad h_L = h_{L_1} + h_{L_2} + h_{L_3}$$

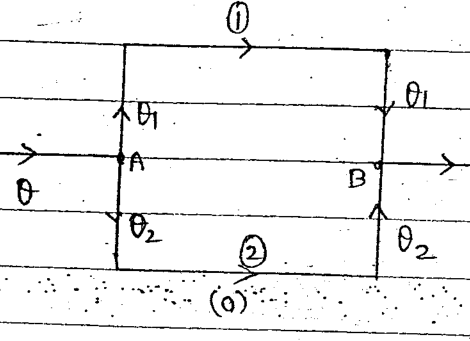
$$= \frac{f l_1 \theta^2}{12 d_1^5} + \frac{f l_2 \theta^2}{12 d_2^5} + \frac{f l_3 \theta^2}{12 d_3^5}$$

$$h_L = h_{le}$$

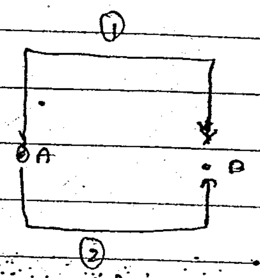
$$\Rightarrow \quad \frac{f l_e \theta^2}{12 d_e^5} = \frac{f l_1 \theta^2}{12 d_1^5} + \frac{f l_2 \theta^2}{12 d_2^5} + \frac{f l_3 \theta^2}{12 d_3^5}$$

$$\Rightarrow \quad \boxed{\frac{l_e}{d_e^5} = \frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5}} \leftarrow \text{Dupuit's Eq}$$

Pipes in Parallel



(a)



(b)

In parallel, b/w A and B Total energy will be same implies head losses will be same

$\theta = \theta_1 + \theta_2$ & $h_{L_1} = h_{L_2}$

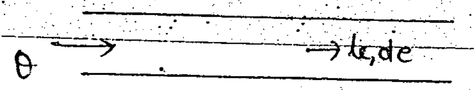
Applying Bernoulli's in fig (b) from the two paths.

$\frac{P_A}{w} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{w} + \frac{V_B^2}{2g} + Z_B + h_{L1}$ — by path ①

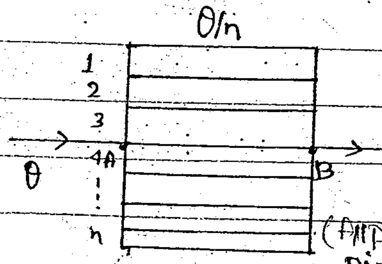
$\frac{P_A}{w} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{w} + \frac{V_B^2}{2g} + Z_B + h_{L2}$ — by path ②

$\Rightarrow h_{L_1} = h_{L2}$

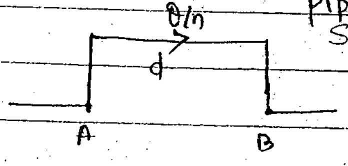
Equivalent Pipe



$h_{L_e} = \frac{f l_e \theta^2}{12 d_e^5}$



(All parallel pipes are similar)



$h_L = f L \left(\frac{\theta}{n}\right)^2 \times \frac{1}{12 d^5}$

$\Rightarrow h_{Le} = h_L \Rightarrow \frac{f L \theta^2}{n^2 \times 12 d^5} = \frac{f l_e \theta^2}{12 d_e^5}$

$\Rightarrow \boxed{\frac{l}{n^2 d^5} = \frac{l_e}{d_e^5}}$ if $l = l_e \Rightarrow \boxed{\begin{array}{l} n^2 d^5 = d_e^5 \\ d_e = n^{2/5} d \end{array}}$

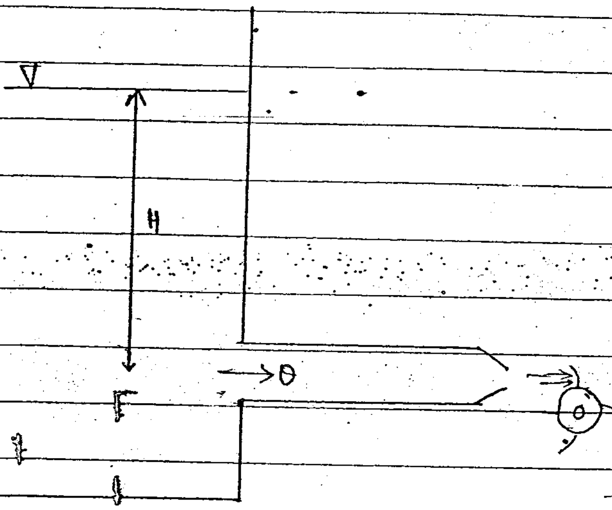
Power Transmission Through Pipes

$P_{th} = W\theta H$

$P_{act} = W\theta(H-h_L)$

$\eta = \frac{P_{act}}{P_{th}} = \frac{\cancel{W\theta}(H-h_L)}{\cancel{W\theta} H}$

$\Rightarrow \eta = \frac{H-h_L}{H}$



$P_{act} = W\theta(H-h_L)$

$P_{act} = W\theta\left(H - \frac{fL\theta^2}{12d^5}\right)$

$\Rightarrow P_{act} = W\left(H\theta - \frac{fL\theta^3}{12d^5}\right)$

for max. power, $\frac{dP_{act}}{d\theta} = 0$

$\Rightarrow W\left[H(1) - \frac{fL3\theta^2}{12d^5}\right] = 0 \Rightarrow H - \frac{3fL\theta^2}{12d^5} = 0$

$\Rightarrow H = \frac{3fL\theta^2}{12d^5}$

$\Rightarrow H = 3h_L \Rightarrow \eta = \frac{H-h_L}{H} = \frac{3h_L - h_L}{L}$

$\Rightarrow \eta_{max} = \frac{2}{3} = 0.667 = 66.67\%$

Q97. Two Water Carrying pipes are Connected in parallel, the length L_1, diameter D_1, friction factor f_1 for the first pipe are 200m, 0.5m, and 0.025, $L_2 = 100m$, $d_2 = 1m$, $f_2 = 0.02$. Then find $\frac{V_2}{V_1}$.

Ans. It is parallel pipe ⇒ head losses will be same.

$\Rightarrow h_{L_1} = h_{L_2}$

$\Rightarrow \frac{f_1 L_1 V_1^2}{\cancel{2g} d_1} = \frac{f_2 L_2 V_2^2}{\cancel{2g} d_2}$

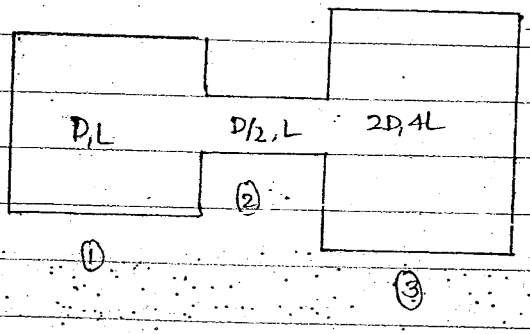
$\Rightarrow \left(\frac{V_2}{V_1}\right)^2 = \frac{f_1 L_1}{d_1} \times \frac{d_2}{f_2 L_2}$

$\Rightarrow \frac{V_2}{V_1} = \sqrt{\frac{f_1 L_1}{d_1} \times \frac{d_2}{f_2 L_2}} = \sqrt{5}$

Q 98 A Compound pipe is shown in fig. then find the equivalent length if the diameter of equivalent pipe is 'D'.

Ans Since the pipes are in Series

$\Rightarrow$ Using Dupuit's eqn

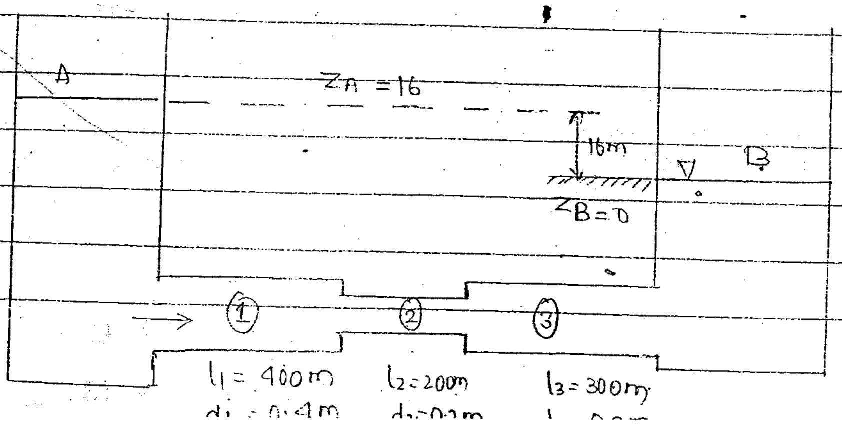


$\Rightarrow \frac{l_e}{d_e^5} = \frac{l_1}{d_1^5} + \frac{l_2}{d_2^5} + \frac{l_3}{d_3^5}$

$\Rightarrow \frac{l_e}{d_e^5} = \frac{L}{D^5} + \frac{L}{(D/2)^5} +$

Q 99 Three pipes are Connected in series as shown in fig. The diff. in Water level b/w two tanks is 16m. If the friction factor for all pipes is same and is equal to 0.02. Then find the discharge through the Compound pipe, neglect minor losses.

Ans.



Reservoir मे velocity हमेशा zero रहेगा।

$$\cancel{\frac{P_A}{w}} + \cancel{\frac{V_A^2}{2g}} + Z_A = \cancel{\frac{P_B}{w}} + \cancel{\frac{V_B^2}{2g}} + Z_B + h_L$$

$$\Rightarrow \quad h_L = 16\,m$$

** When reservoirs are Connected in Series then head loss will be equal to head difference in reservoirs.

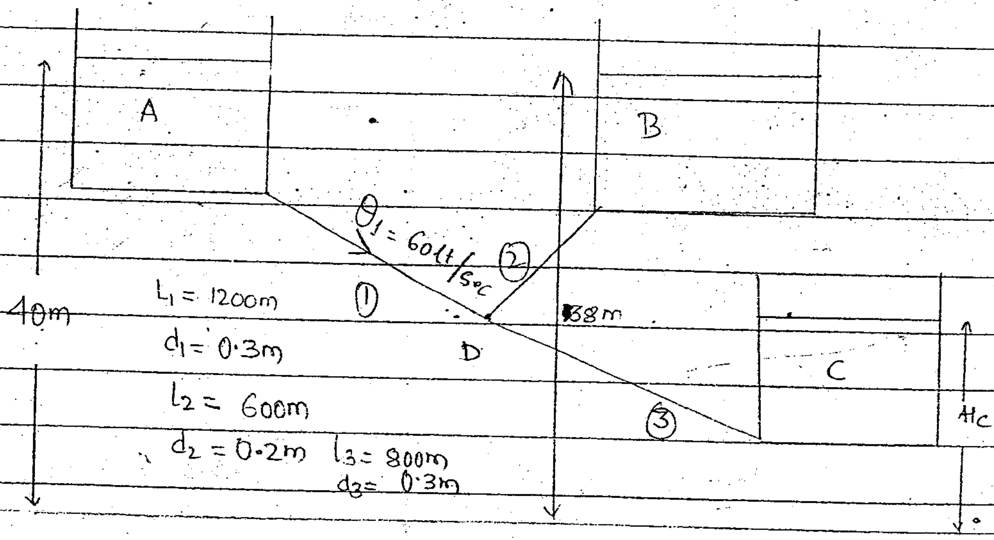
$$\Rightarrow \quad h_{L_1} + h_{L_2} + h_{L_3} = 16$$

$$\Rightarrow \quad \frac{fL_1Q^2}{12d_1^5} + \frac{fL_2Q^2}{12d_2^5} + \frac{fL_3Q^2}{12d_3^5} = 16$$

On Substituting Values

$$Q = 0.11\ m^3/sec$$

Q100. Three reservoir A, B and C are Connected by a pipe system as shown in fig. find the discharge into and from reservoirs B and C if flow rate from reservoir A is 60 lt/s. find the height of water level. find the height of water level in reservoir 'C'. Take friction factor as 0.024 for all pipes.



Ans. For reservoirs, $\frac{\cancel{P_A}^{0}}{w} + \frac{\cancel{V_A^2}^{0}}{2g} + Z_A = H_A$

⇒ $H_A = Z_A$ (i.e. Total head for the reservoir is equal to potential head).

Applying Bernoulli's eqn b/w A and D

$$\Rightarrow \frac{P_A}{w} + \frac{V_A^2}{2g} + Z_A = \frac{P_D}{w} + \frac{V_D^2}{2g} + Z_D + h_{L1}$$

$$\Rightarrow H_A = H_D + h_{L1}$$

$$\Rightarrow 40 = H_D + \frac{f_1 L_1 Q_1^2}{12 d_1^5}$$

$$\Rightarrow 40 = H_D + \frac{0.024 \times 1200 \times (60\times10^{-3})^2}{12\times(0.3\cancel{5})^5}$$

$$\Rightarrow H_D = 36.4 \text{ m}$$

as $H_B > H_D$ ∴ flow is from 'B' to 'D'

Since, dirxn is known so applying Bernoulli's eqn

$$\Rightarrow H_B = H_D + h_{L2}$$

$$\Rightarrow 38 = 36.4 + \frac{0.024 \times 600 \times Q_2^2}{12(0.2)^5}$$

$$\Rightarrow Q_2 = 20.3\times10^{-3} \text{ m}^3/\text{sec}.$$

Since discharge is coming out of A and B, so it will have to flow towards C i.e. D to C.

$$Q_1 + Q_2 = Q_3$$

$$\Rightarrow 60\times10^{-3} + 20.3\times10^{-3} = Q_3 = 80.3\times10^{-3} \text{ m}^3/\text{sec}$$

Applying Bernoulli's eqn b/w D and C,

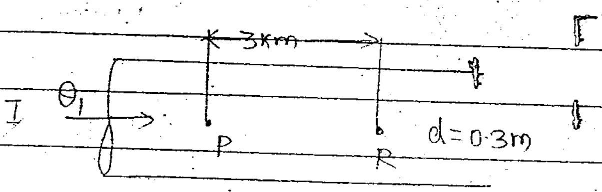
$$H_D = H_C + h_{L3}$$

$$\Rightarrow 36.4 = H_C + \frac{f_3 L_3 Q_3^2}{12 d_3^5} \Rightarrow 36.4 = H_C + \frac{0.024\times(800\times(80.3\times10^{-3})^2}{12\times(\underset{0.3}{\cancel{0.35}})^5}$$

$$\Rightarrow H_C = 32.15 \text{m}$$

Q101 A pipeline of 0.3m dia. & 3km length carries water from point 'P' to point 'R' as shown in fig. The piezometric heads at 'P' and 'R' are to be maintained at 100m and 80m resp. To increase the discharge a 2nd pipe is added in parallel to the existing pipe from 'Q' to 'R'. The length of additional pipe is also 2km. Assume friction factor 'f' is equal to 0.04 for all pipes and ignore minor loss. Calculate the % increase in discharge if the additional pipe ~~of d~~ has a dia. of 0.3m.

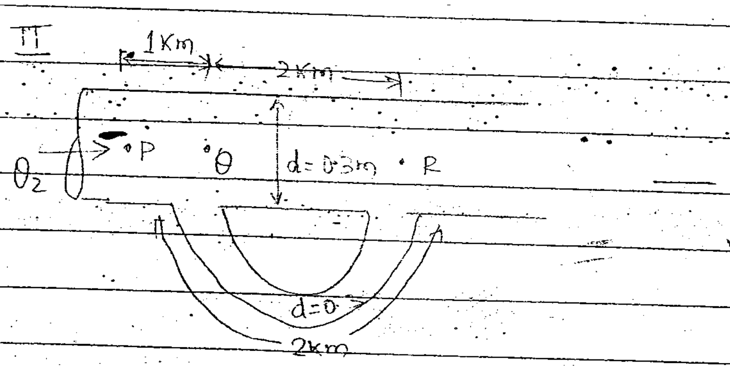
Ans. (I) head loss in pipe is the frictional head loss



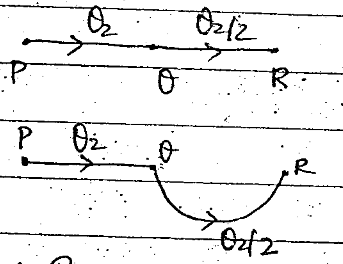
$$\Rightarrow 100-80 = h_L = \frac{fLQ^2}{12d^5}.$$

$$\Rightarrow 20 = \frac{0.04\times3000\times Q_1^2}{12\times(0.3)^5}$$

$$\Rightarrow Q_1 = 0.069\ m^3/sec.$$



(II)



Taking any one of them,

$$\Rightarrow 100-80 = h_L = h_{LPQ} + h_{LQR}$$

$$\Rightarrow 20 = \frac{fL_{PQ}Q_{PQ}^2}{12d_{PQ}^5} + \frac{fL_{QR}Q_{QR}^2}{12d_{QR}^5}$$

$$\Rightarrow 20 = \frac{f}{12d^5}\left[L_{PQ}Q_{PQ}^2 + L_{QR}\times Q_{QR}^2\right]$$

$$\Rightarrow \quad 20 = \frac{0.04}{12(0.3)^5}\left[1000(\theta_2)^2 + 2000\left(\frac{\theta_2}{2}\right)^2\right]$$

$$\Rightarrow \quad \theta_2 = 0.0985 \ m^2/sec$$

∴ % increase in discharge

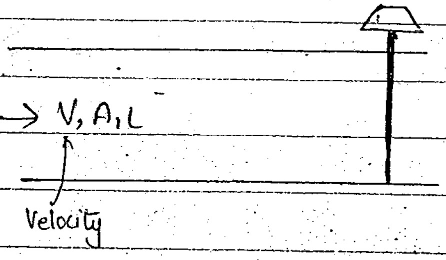
$$= \frac{\theta_2 - \theta_1}{\theta_1} \times 100 = \frac{0.0985 - 0.069}{0.069} \times 100 = 42.7\%$$

Water Hammering in Pipes

When liquid flows through pipe and if the valve is closed suddenly high pressure wave is generated this causes huge noise known as knocking or water hammering. This pressure wave can even burst the pipe.

Case (i) Gradual closure of valve

Principle - Retarding force is equal to pressure generation force.



$$ma = \text{Retarding force}$$

$$\text{Retarding force} = \rho AL\left(\frac{V-0}{t}\right)$$

$$\Rightarrow \text{Retarding force} = \frac{\rho ALV}{t}$$

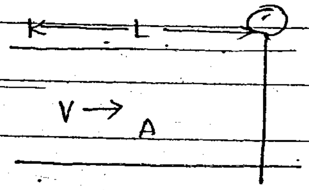
$$\text{Pressure generation force} = PA$$

$$\therefore \text{Retarding force} = \text{pressure generation force}$$

$$\Rightarrow PA = \frac{\rho ALV}{t} \quad \Rightarrow P = \frac{\rho LV}{t}$$

Case(ii) Sudden Closure of Valve and the pipes is Rigid.

Rigid - pipe is not absorbing any strain energy.



∴ the principle is that

Loss in K.E = Energy stored in Water

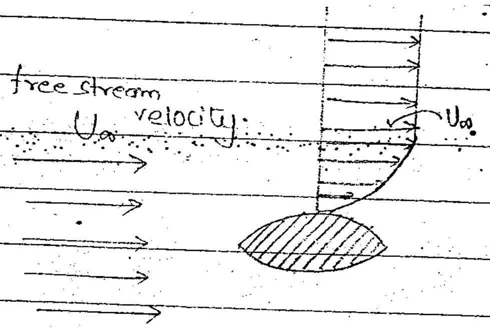
$$\Rightarrow \boxed{\frac{1}{2}\rho ALV^2 = \frac{P^2}{2K} \times Vol}$$

$$\Rightarrow \frac{1}{\cancel{2}}\rho \cancel{AL}V^2 = \frac{P^2}{\cancel{2}K} \times \cancel{AL}$$

$$\Rightarrow \rho V^2 = \frac{P^2}{K} \Rightarrow P = \sqrt{\rho K}\ V$$

BOUNDARY LAYER THEORY

Boundary Layer Region में Bernoulli's Eqn नहीं लगता as due to viscosity existing in Boundary layer and out side Boundary layer it is non-viscous and Bernoulli's eqn can be applied.

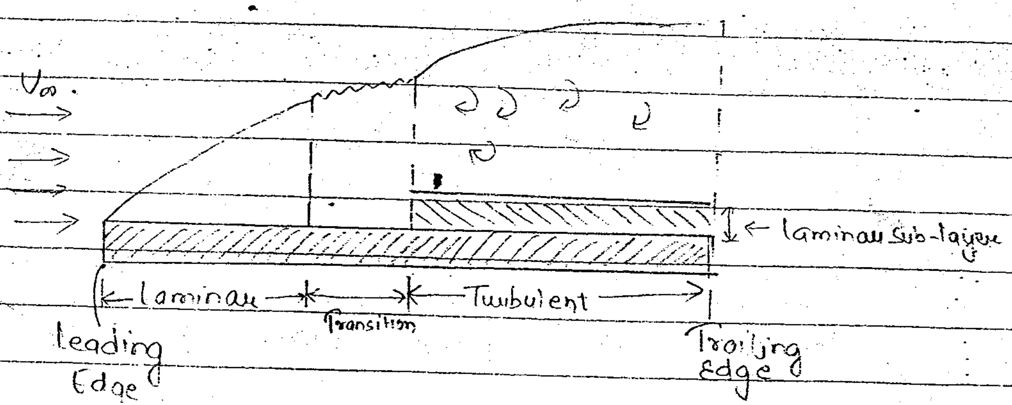


When a real fluid flows past a solid-boundary due to no-slip at the boundary, the velocity of fluid will be same as that of the boundary. If the boundary is at rest the fluid will also have zero velocity. Away from the boundary the velocity increases and at some distance it reaches free stream velocities. This region in which there are velocity gradient is known as boundary layer region.

The flow in boundary layer region is viscous in nature.

Development of boundary layer over a flat Plate

In external flow as pressure is everywhere atmospheric so there is no pressure drop along the length of x-s/c.



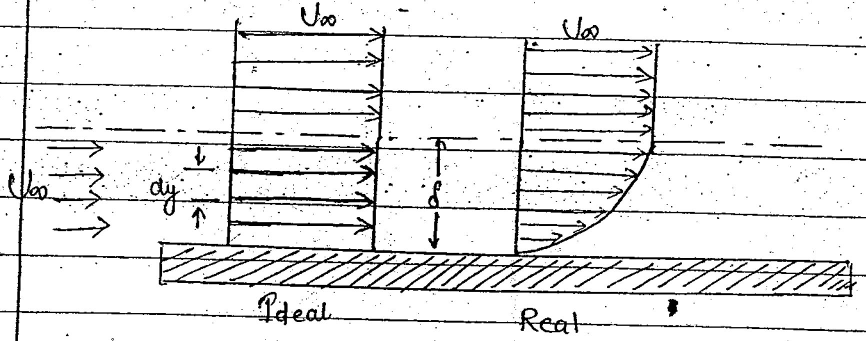
When a real fluid flows past the flat plate, the velocity at the leading edge is zero and the retardation of fluid particles increases as more area of the plate is exposed to the flow and hence boundary layer thickness increases as the distance from the leading edge increases. Upto certain distance from the leading edge the flow in the boundary layer is found to be laminar. As laminar B.L. grows instability occurs and flow changes from laminar to turbulent through transition. It is found that even in turbulent boundary layer region closed to the plate, the flow is laminar and this layer is known as laminar sublayer region and this exists in turbulent boundary layer region.

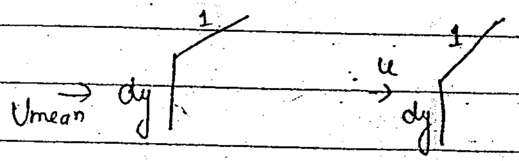
Boundary Layer Thickness or Nominal Boundary Layer Thickness (δ)

It is the distance from boundary to the point in y dirn, where the velocity is 99% of free stream velocity. (This is only arbitrary definition and has no mathematical analysis)

for all calculation purposes at $y = \delta$, $u = U_\infty$

Displacement Thickness (δ^*)





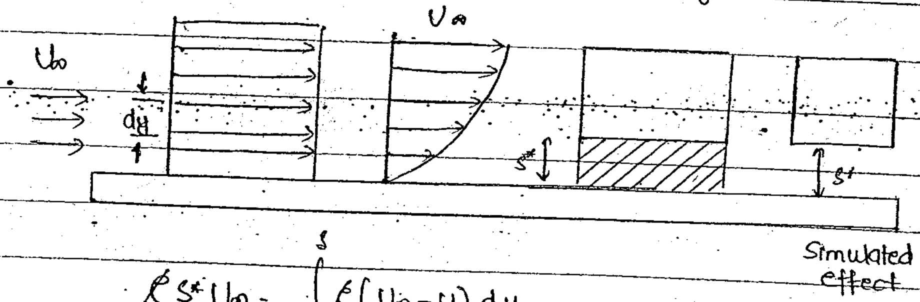
$$\dot{m}_{ideal} = \rho \cdot dy \cdot 1 \times U_{mean} \qquad \dot{m}_{real} = \rho u dy$$

$$\dot{m}_{ideal} = \rho U_\infty dy$$

Decrease in mass flow rate due to B.L growth = $\dot{m}_{ideal} - \dot{m}_{real}$

$$= \rho U_\infty dy - \rho u dy = \rho(U_\infty - u)dy$$

Total reduction in mass flow rate $= \int_0^\delta \rho(U_\infty - u)\,dy$



$$\rho \delta^* U_\infty = \int_0^\delta \rho(U_\infty - u)\,dy$$

$$\Rightarrow \delta^* = \int_0^\delta \left(\frac{U_\infty - u}{U_\infty}\right) dy$$

$$\Rightarrow \boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy}$$

It is the distance by which boundary should be displaced in order to compensate for reduction in mass flow rate due to boundary layer growth.

Momentum Thickness (θ)

It is the distance by which the boundary should be displaced in order to compensate for the momentum due to boundary layer growth

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

Energy Thickness (δ_E)

It is the distance by which the boundary should be displaced in order to compensate for the kinetic energy due to boundary layer growth.

$$\delta_E = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u^2}{U_\infty^2}\right) dy$$

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**** Boundary Cond^ns

- At $x=0$, $\delta=0$
- At $y=0$, $u=0$
- At $y=\delta$, $u=u_\infty$
- At $y=\delta$, $\frac{du}{dy}=0$

Wall shear stress $\tau_0 = \mu \frac{du}{dy}\Big|_{y=0}$

Q102. The velocity distribution in boundry layer is given by $\frac{u}{u_\infty} = \frac{y}{\delta}$, then find displacement thickness and momentum thickness.

Ans. $\frac{u}{U_\infty} = \frac{y}{\delta}$ {velocity profile}

$$\Rightarrow \delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

$$\Rightarrow \delta^* = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy$$

$$\Rightarrow \delta^* = \int_0^\delta \left(dy - \frac{y\,dy}{\delta}\right) \Rightarrow \delta^* = \left(y - \frac{y^2}{2\delta}\right)_0^\delta$$

$$\Rightarrow \delta^* = \left[\delta - \frac{\delta^2}{2\delta}\right] \Rightarrow \delta^* = \frac{\delta}{2}$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

$$\Rightarrow \theta = \int_0^\delta \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy = \int_0^\delta \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy$$

$$\Rightarrow \theta = \left[\frac{y^2}{2\delta} - \frac{y^3}{3\delta^2}\right]_0^\delta = \frac{\delta}{6}$$

** Shape factor (H) = $\frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6} = 3$

Q103. Assume that the shear stress distribution varies linearly in a laminar boundary layer such that $\tau = \tau_0\left(1 - \frac{y}{\delta}\right)$. find the displacement thickness and momentum thickness.

Ans $\tau = \mu \frac{du}{dy}$

$\Rightarrow \mu \frac{du}{dy} = \tau_0\left(1 - \frac{y}{\delta}\right)$

$\Rightarrow \frac{du}{dy} = \frac{\tau_0}{\mu}\left(1 - \frac{y}{\delta}\right) \Rightarrow du = \frac{\tau_0}{\mu}\left(1 - \frac{y}{\delta}\right)dy$

$\Rightarrow u = \frac{\tau_0}{\mu}\left(y - \frac{y^2}{2\delta}\right) + c$

at $y = 0 \Rightarrow u = 0$

$\Rightarrow 0 = 0 + c \Rightarrow c = 0$

$\Rightarrow u = \frac{\tau_0}{\mu}\left(y - \frac{y^2}{2\delta}\right)$ —— ①

at $y = \delta$, $u = U_\infty$

$\Rightarrow U_\infty = \frac{\tau_0}{\mu}\left(\delta - \frac{\delta^2}{2\delta}\right) = \frac{\tau_0}{2\mu}\delta$ —— ②

$$\Rightarrow \frac{u}{U_\infty} = \frac{\frac{\tau_0}{\mu}\left(y - \frac{y^2}{2\delta}\right)}{\frac{\tau_0}{2\mu}\delta} = \frac{2}{\delta}\left(y - \frac{y^2}{2\delta}\right)$$

$$\Rightarrow \frac{u}{U_\infty} = \frac{2y}{\delta} - \frac{2y^2}{2\delta^2} = \frac{2y}{\delta} - \frac{y^2}{\delta^2}$$

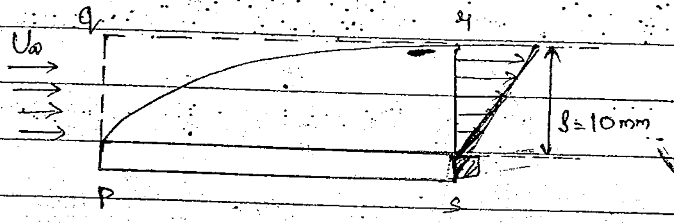
$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy$$

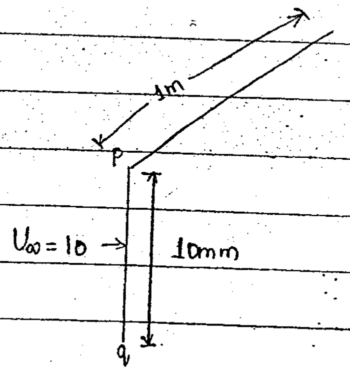
$$\Rightarrow \delta^* = \int_0^\delta \left(1 - \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\right) dy$$

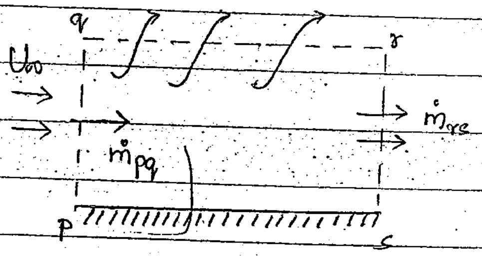
$$\Rightarrow \delta^* = \frac{\delta}{3} \quad \text{and} \quad \theta = \frac{2\delta}{15}$$

Q104 ** A smooth flat is placed in a gas stream flowing at a free stream velocity of 10m/s. The thickness of the boundary layer at rs is 10mm, the width of the plate is 1m and density of the gas is 1kg/m³. The velocity distribution is $u = U_\infty\left(\frac{y}{\delta}\right)$ at the section 'rs', where y is distance from the plate, find the mass flow rate across 'qr' and also find the drag force on the plate b/w 'ps'.

Ans. Since there is reduction in mass flow rate due to formation of B.L so, some fluid will come out from top of the control volume.



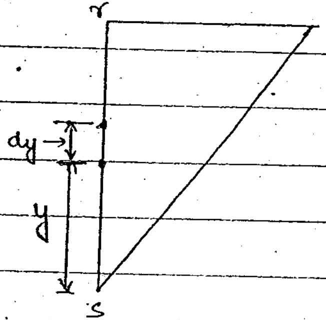




$$\dot{m}_{pq} = \rho A U_\infty = 1 \times 10^{-3} \times 1 \times 10$$

$$\Rightarrow \dot{m}_{pq} = 0.1 \text{ kg/sec}$$

$u = \frac{U_\infty y}{\delta}$



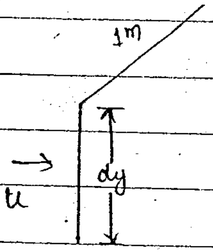
$m_{HS} = \int_0^\delta \rho u dy$

$\Rightarrow m_{HS} = \int_0^\delta \rho U_\infty \frac{y}{\delta} dy$

$\Rightarrow m_{HS} = \frac{\rho U_\infty}{\delta}\left(\frac{y^2}{2}\right)_0^\delta = \frac{\rho U_\infty \times \delta^2}{2\delta}$

$\Rightarrow m_{HS} = \frac{\rho U_\infty \delta}{2} = \frac{1 \times 10 \times 10 \times 10^{-3}}{2}$

$\Rightarrow m_{HS} = 0.05 kg/sec$

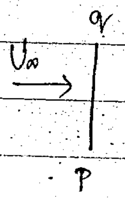


$\dot{m}_{pq} \mp \dot{m}_{qr} = \dot{m}_{HS}$

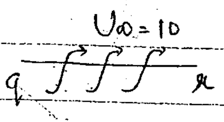
$\Rightarrow \dot{m}_{qr} = \dot{m}_{pq} - \dot{m}_{HS}$

$\Rightarrow \dot{m}_{qr} = 0.1 kg/sec - 0.05 kg/sec = 0.05 kg/sec$

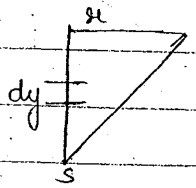
According to Newton's 2nd law of motion, rate of change of momentum is equal to force ∴ Drag force (force exerted by fluid in the dirxn parallel to the motion) is equal to (entry momentum rate − exit momentum rate)

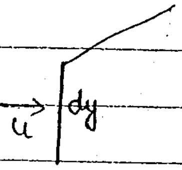


momentum entering through Pq (P_{pq}) $= 0.1 \times 10 = 1N$



momentum exiting through qr (P_{qr}) $= 10 \times 0.05 = 0.5N$





$\Rightarrow \dot{m}_{HS} = \rho \times u \times dy \times 1$

$\Rightarrow P_{HS} = \int_0^\delta \rho \times dy \times 1 \times u^2$

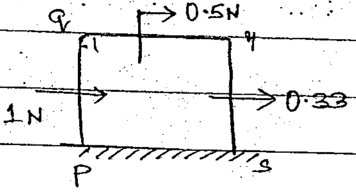
$$\Rightarrow P_{HS} = \int_0^\delta \rho u^2 \, dy$$

$$\Rightarrow P_{HS} = \int_0^\delta \frac{\rho U_\infty^2 \times y^2 dy}{\delta^2} = \frac{\rho U_\infty^2}{\delta^2}\left[\frac{y^3}{3}\right]_0^\delta$$

$$\Rightarrow P_{HS} = \frac{\rho U_\infty^2}{3\delta^2}\left(\delta^3\right) \longrightarrow P_{HS} = \frac{\rho U_\infty^2 \times \delta}{3}$$

$$\Rightarrow P_{HS} = \frac{1\times10^2 \times (10\times10^{-3})}{3} = 0.333N$$

$$\Rightarrow \text{Drag force } F_D = 1-(0.5+0.333)$$
$$= 0.17N$$



Von-Karman Momentum Integral Eqn (for flat plate)

Assumptions

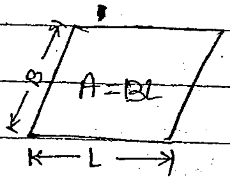
- Steady flow
- 2-Dimensional flow
- Incompressible flow
- $\frac{dP}{dx} = 0$ (for flatplates but exists for internal flow)

$$\boxed{\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx}}$$

Where, θ is momentum thickness
x is distance from leading edge
τ_0 is shear stress on the surface of plate.

Average Drag Coefficient ~~(C_D)~~ (C_D)

$$C_D = \frac{F_D}{\frac{1}{2}\rho A V_\infty^2}$$



With the help of C_D, the average drag force F_D can be found

Local Drag Coefficient or local Skin friction Coefficient (C_{fx})

$$C_{fx} = \frac{\tau_0}{\frac{1}{2}\rho U_\infty^2} \quad \{\text{Reynold's - Colburn Analogy}\}$$

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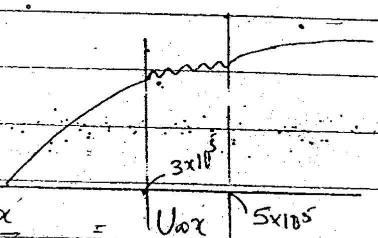
Reynold's Number :-



$$Re_x = \frac{\rho U_\infty x}{\mu} = \frac{\rho U_\infty x}{\mu} = \frac{U_\infty x}{(\mu/\rho)} = \frac{U_\infty x}{\nu}$$

Where 'x' is the distance from the leading edge.
i.e. laminar flow exists upto transition state and $Re = 5 \times 10^5$

Q105. For a velocity profile for a laminar Boundary layer

$$\frac{u}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{y^3}{2\delta^3}.$$

Find (a). Boundary Layer Thickness.
(b). Shear stress on Surface of plate
(c). Drag force
(d). Average Drag Coefficient in terms of Reynold's no.

Ans.

$$\frac{u}{U_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{y^3}{2\delta^3}$$

By von-Karman Eq^n,

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{d\theta}{dx}$$

$$\theta = \int_0^\delta \frac{u}{U_\infty}\left(1 - \frac{u}{U_\infty}\right) dy$$

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$$\theta = \int_0^\delta \left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right)\left(1 - \left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right)\right) dy \Rightarrow \theta = \frac{39\delta}{280}$$

$$\frac{\tau_0}{\rho U_\infty^2} = \frac{39}{280} \frac{d\delta}{dx}$$

$$\Rightarrow \tau_0 = \frac{39}{280} \rho U_\infty^2 \frac{d\delta}{dx} \text{ ——— } ①$$

$$\frac{u}{U_\infty} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$$

$$\Rightarrow \frac{du}{dy} = U_\infty\left[\frac{3}{2\delta} - \frac{3y^2}{2\delta^3}\right]$$

$$\left.\frac{du}{dy}\right|_{y=0} = \frac{3U_\infty}{2\delta} \Rightarrow \tau_0 = \mu \left.\frac{du}{dy}\right|_{y=0}$$

$$\Rightarrow \tau_0 = \mu \left.\frac{du}{dy}\right|_{y=0} \Rightarrow \tau_0 = \mu \times \frac{3U_\infty}{2\delta} \text{ ——— } ②$$

from ① and ②

$$\frac{\overset{13}{\cancel{39}}}{\underset{140}{\cancel{280}}} \rho U_\infty^{\cancel{2}} \frac{d\delta}{dx} = \frac{\cancel{3}\mu \cancel{U_\infty}}{\cancel{2}\delta}$$

$$\Rightarrow \frac{13}{140} \frac{\rho U_\infty}{\mu} \delta \cdot d\delta = dx$$

on Integrating,

$$\frac{13}{140} \frac{\rho U_\infty}{\mu} \frac{\delta^2}{2} + C = x$$

At $x=0$, $\delta=0 \Rightarrow C=0$

$$\Rightarrow \frac{13}{280} \times \frac{\rho V_\infty}{\mu}\delta^2 = x$$

$$\Rightarrow \cancel{\frac{13}{280} \times \frac{\rho V_\infty \delta^2}{\mu} = x}$$

$$\Rightarrow \frac{13}{280} \times \frac{\rho V_\infty x x}{\mu}\delta^2 = x \cdot x$$

$$\Rightarrow \frac{13}{280} \times Re_x \delta^2 = x^2$$

$$\Rightarrow \delta^2 = \frac{280}{13}\frac{x^2}{Re_x} \Rightarrow \boxed{\delta = \frac{4.64\,x}{\sqrt{Re_x}}}$$

however we have derived it for a particular velocity profile but the expression is similar except the constant.

$$\delta = \frac{4.64\,x}{\sqrt{Re_x}} \Rightarrow \delta = \frac{4.64x}{\sqrt{\frac{\rho V_\infty x}{\mu}}} \Rightarrow \delta \propto \frac{x}{\sqrt{x}}$$

$$\Rightarrow \boxed{\delta \propto \sqrt{x}} \Rightarrow \boxed{\frac{\delta_1}{\delta_2} = \frac{\sqrt{x_1}}{\sqrt{x_2}}}$$ ← Laminar Boundary layer over flat plates.

(b) $\tau_0 = \frac{3\mu V_\infty}{2\delta}$ —— ②

→ τ_0 is decreasing

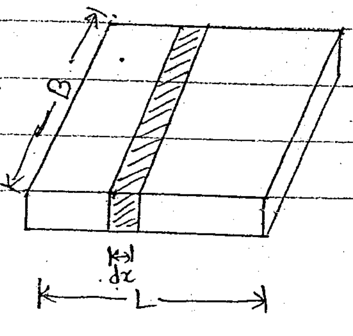
$$\Rightarrow \tau_0 = \frac{3\mu V_\infty}{\frac{2\times 4.64\times x}{\sqrt{Re_x}}} = \frac{0.323\,\mu V_\infty\sqrt{Re_x}}{x}$$

$$\Rightarrow \tau_0 \propto \frac{\sqrt{x}}{x} \Rightarrow \tau_0 \propto \frac{1}{\sqrt{x}} \Rightarrow \boxed{\frac{\tau_{01}}{\tau_{02}} = \frac{\sqrt{x_2}}{\sqrt{x_1}}}$$

$$dF_D = \tau_0 B\,dx$$

$$\Rightarrow F_D = \int_0^L \tau_0 B\,dx$$

$$\Rightarrow F_D = \int_0^L \frac{0.323\,\mu V_\infty\sqrt{Re_x}}{x} B\,dx$$



$f_D = 0.646\, \beta \mu U_\infty \sqrt{Re_L}$

$C_D = \dfrac{F_D}{\frac{1}{2}\rho A U_\infty^2} = \dfrac{0.646\, \cancel{B} \mu \cancel{U_\infty} \sqrt{Re_L}}{\frac{1}{2}\rho \cancel{B} L U_\infty^2}$

$C_D = \dfrac{1.292\, \mu \sqrt{Re_L}}{\rho U_\infty L} = \dfrac{1.292 \sqrt{Re_L}}{Re_L}$

$\Rightarrow \boxed{C_D = \dfrac{1.292}{\sqrt{Re_L}}}$

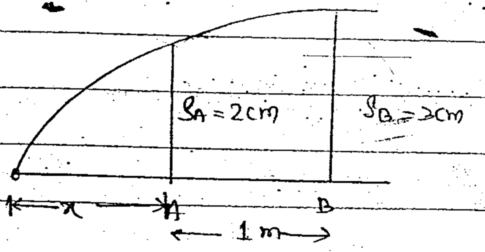
Q106 The thickness of Laminar boundary layer on a flat plate at a point A is 2cm and at a point B, 1m from A the boundary layer thickness is 3cm then what is the distance of A from leading edge.

Ans. $x_A = x, \quad \delta_A = 2$

$x_B = x+1 \quad \delta_B = 3.$

$\Rightarrow \dfrac{\delta_A}{\delta_B} = \sqrt{\dfrac{x}{x+1}} \Rightarrow \dfrac{x}{x+1} = \dfrac{4}{9}$

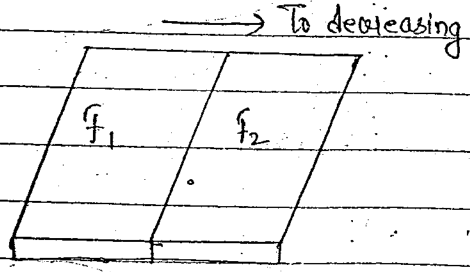
$\Rightarrow x = 0.8$ cm



Q107. Consider an incompressible boundary layer over a flat plate of length 'L' along the dirxn of incoming free stream. If 'F' is ratio of drag force on 1st half of the plate to the drag force on the plate.

Ans. $F = \dfrac{F_1}{F_2}$ as $f_1 > F_2$

$\Rightarrow \dfrac{f_1}{f_2} > 1$



Q108. For air flow over a flat plate the velocity 'u' and boundary layer thickness 'δ' can be expressed as

$$\frac{u}{V_\infty} = \cancel{\frac{2y}{2\delta}}\ \frac{3}{2}\frac{y}{\delta} - \frac{y^3}{2\delta^3} \quad \text{and} \quad \delta = \frac{4.64\,x}{\sqrt{Re_x}}$$

free stream velocity = 2m/s and $\nu = 1.5 \times 10^{-5}\,m^2/s$ then find the shear stress on the surface of the plate at $x = 1$. Take density of air as 1.2kg/m3.

Ans

$$\frac{u}{V_\infty} = \frac{3}{2}\frac{y}{\delta} - \frac{y^3}{2\delta^3}$$

$$\Rightarrow \frac{du}{dy} = V_\infty\left(\frac{3}{2\delta} - \frac{3y^2}{2\delta^3}\right)$$

$$\left.\frac{du}{dy}\right|_{y=0} = \frac{3V_\infty}{2\delta}$$

$$\Rightarrow \tau = \mu\frac{3V_\infty}{2\delta}$$

$$\Rightarrow \delta = \frac{4.64 \times 1}{\sqrt{\frac{\times 2 \times x\, 10^5 \times 1}{1.5}}} = 0.012\,m$$

$$\Rightarrow \tau = \frac{1.5 \times 10^{-5} \times 1.2 \times 3 \times 2}{2 \times 0.012} = 4.5 \times 10^{-3}\,N/m^2$$

Blausius Equation

	Laminar	Turbulent
for flat plates	$\delta = 5x/\sqrt{Re_x}$	$\delta = \frac{0.576x}{(Re_x)^{1/5}}$
When velocity profile is not given.	$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$	$C_{fx} = \frac{0.059}{(Re_x)^{1/5}}$
	$C_D = \frac{1.328}{\sqrt{Re_L}}$	$C_D = \frac{0.074}{Re_L^{1/5}}$

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Q109. Calculate the drag force on the plate 0.15m wide and 0.45m long placed in a stream of oil having a free stream velocity of 6 m/s. Also find the thickness of boundary layer and shear stress at the trailing edge. Take density of oil as ~~901~~ 925 kg/m³ and kinematic viscosity = 9×10^{-5} m²/s.

Ans.

$L = 0.45$

$b = 0.15$

$A = 0.0675 m^2$

$$\Rightarrow Re_L = \frac{\rho U_\infty L}{\mu} = \frac{\cancel{925} \times 6 \times 0.45}{\cancel{925} \times 9 \times 10^{-5}} \doteq 3 \times 10^4$$

$\Rightarrow$ the flow is laminar.

$$C_D = \frac{1.328}{\sqrt{Re_L}} = \frac{\cancel{\tau_0}\, F_D}{\frac{1}{2}\rho U_\infty^2 \times A}$$

$$\Rightarrow \frac{1.328}{\sqrt{3} \times 100} = \frac{2 F_D}{925 \times 6^2 \times 0.0675}$$

$$\Rightarrow F_D = 8.6\ N$$

∴ ~~Total F_D = 2×F_D (i.e. on both sides)~~

~~= [illegible]~~

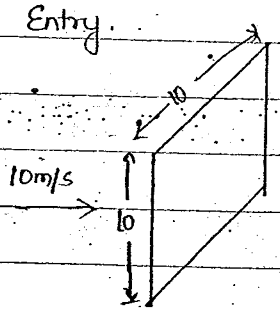
$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 0.45}{\sqrt{3} \times 100} = 0.0129 = 12.99 mm$$

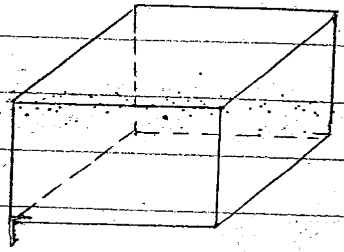
$$C_{fx} = \frac{0.664}{\sqrt{Re_x}} = \frac{\tau_0}{\frac{1}{2}\rho U_\infty^2}$$

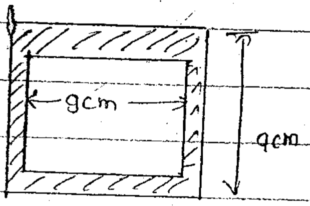
$$\Rightarrow \frac{0.664}{\sqrt{30000}} = \frac{2\tau_0}{925 \times 36} \Rightarrow \tau_0 = 63.8\ N/m^2$$

Q110 ✓ ****
Air flows in a square duct of 10cm side, at the entrance, boundary layer thickness is negligible and its velocity is 10m/s. At the exit the displacement thickness is 5mm on each wall, then find the velocity outside the boundary layer at the exit.

Ans. At Entry.







$A_1 = 10 \times 10 \Rightarrow A_1 = 100\,cm^2$

$\Rightarrow V_1 = 10\,m/s$

Since there is no accumulation of mass 12:35

$\Rightarrow$ Continuity eqn can be applied

$$\Rightarrow A_1V_1 = A_2V_2$$

$$\Rightarrow 100 \times 10 = (9)^2 \times V_2$$

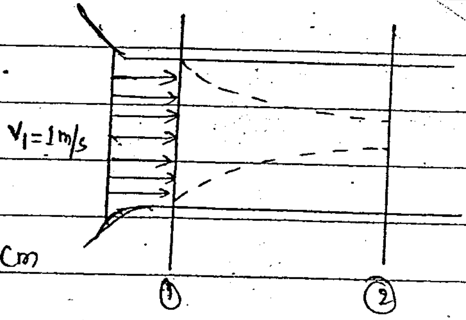
$$\Rightarrow \underline{\underline{V_2 = 12.3456\ m/s}}\ \text{Ans}$$

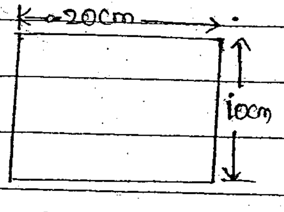
Q111. ✓
Air enters a horizontal duct of cross-section 20cm×10cm at a uniform velocity of 1m/s as shown in fig. Boundary layer starts to grow on all four walls. The boundary layer thickness δ was measured to be 4cm on each of 4 walls. If the displacement thickness (δ^*) is $\frac{\delta}{8}$. Find the velocity outside the boundary layer at section 2 and also find the pressure difference b/w s/c ① and s/c ②. Take the density as 1.15 kg/m³. Assume steady incompressible flow and neglect viscous losses outside the boundary layer.

Ans.

$V_1 = 1 m/s$

$A = 20 \times 10 = 200 cm^2$





Entry area

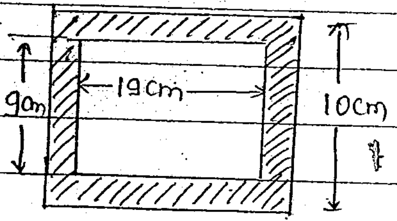
$\delta^* = \frac{\delta}{8} = \frac{4}{8} = 0.5 cm$

$\Rightarrow A_2 = 19 \times 9 = 171 cm^2$

$\Rightarrow A_1 V_1 = A_2 V_2$

$\Rightarrow 200 \times 1 = 171 \times V_2$

$\Rightarrow V_2 = 1.169 m/s$



Area of exit

Applying Bernoulli's eqn (Applied outside the boundary layer)

$$\Rightarrow \frac{P_1}{w} + \frac{V_1^2}{2g} = \frac{P_2}{w} + \frac{V_2^2}{2g}$$

$$\Rightarrow \frac{P_1 - P_2}{w} = \frac{V_2^2 - V_1^2}{2g}$$

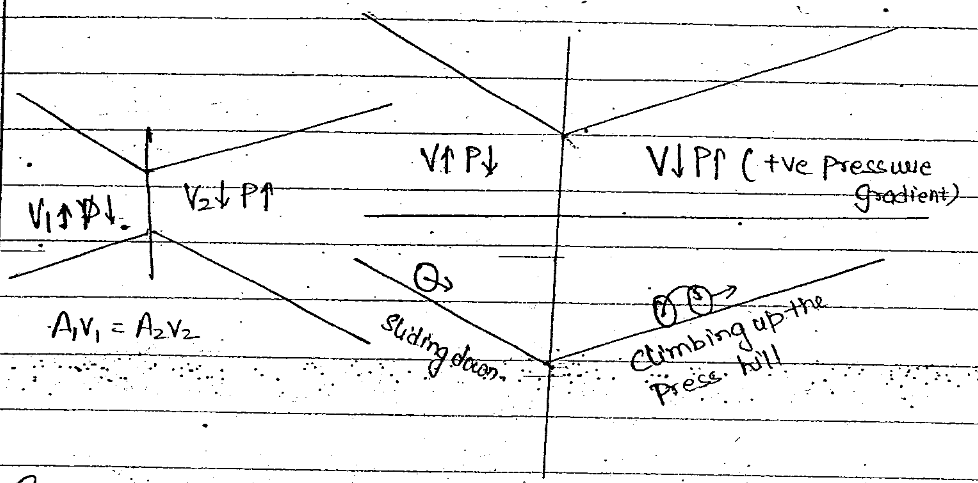
$$\Rightarrow \frac{P_1 - P_2}{\rho} = \frac{V_2^2 - V_1^2}{2} \Rightarrow P_1 - P_2 = \frac{\rho}{2}(V_2^2 - V_1^2)$$

$$\Rightarrow P_1 - P_2 = \frac{1.15}{2}\left[1.169^2 - 1\right]$$

$$\Rightarrow P_1 - P_2 = 0.27 N/m^2$$

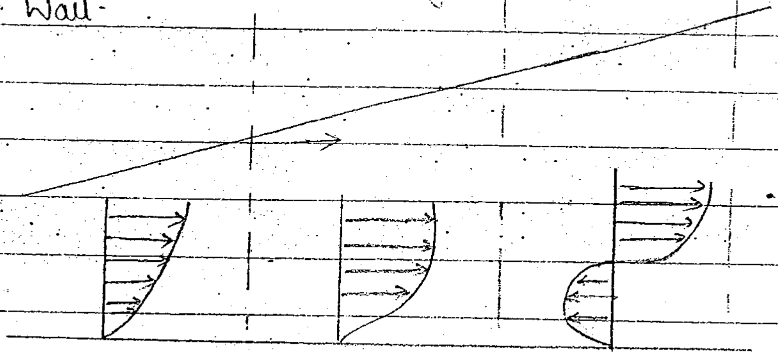
Boundary Layer Separation

When fluid flows in a converging passage, velocity increases and pressure decrease this flow is known as accelerated flow.



In case of divergent passage velocity decreases and pressure increases this flow is known as decelarated flow i.e. the fluid moves under the +ve pressure gradients. If the velocity reduction is more, at some point of time the momentum may not support the flow and hence the flow may reverse its dir^n from the boundary and this is known as boundary layer separation and this occurs under +ve pressure gradients and these positive pressure gradients are known as adverse pressure gradients.

As separation occurs at boundary wall so velocity gradients are found at wall.



$\left.\frac{du}{dy}\right|_{y=0} = +ve$ | $\left.\frac{du}{dy}\right|_{y=0} = 0$ | $\left.\frac{du}{dy}\right|_{y=0} = -ve$

no separation (Attached flow)	About to separate (Separation point)	(Separated flow.)

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** At the Separation point the Shear stress is zero,

as $\frac{du}{dy} = 0$

DIMENSIONAL ANALYSIS

Rayleigh's Method of Grouping

For a laminar flow in a pipe the pressure drop ΔP is a fx^n of pipe length l, Diameter D, Velocity V and Viscosity μ. Using Rayleigh method, develop the expression for ΔP.

$$P = \frac{F}{A} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

$$\Delta P = f(L, D, V, \mu)$$

$$\Delta P = K \cdot L^a D^b V^c \mu^d$$

$$\Rightarrow ML^{-1}T^{-2} = L^a \cdot L^b (LT^{-1})^c (ML^{-1}T^{-1})^d$$

$$\Rightarrow ML^{-1}T^{-2} = M^d \cdot L^{a+b+c-d} \, T^{-c-d}$$

$\Rightarrow$ $d = 1$,

$a+b+c-d = -1$

$-c-d = -2$

$\Rightarrow c+d = 2$

$\Rightarrow c+1 = 2 \quad \Rightarrow c = 1$

$a+b+c-d = -1 \quad \Rightarrow a+b+1-1 = -1$

$\Rightarrow a+b = -1$

$b = -1-a$

$$\Rightarrow \Delta P = KL^a D^{-1-a} V^1 \mu^1$$

$$\Rightarrow \Delta P = \frac{KL^a}{D^{1+a}} \cdot \mu V$$

$$\Rightarrow \Delta P = \frac{K\mu V}{D}\left(\frac{L}{D}\right)^a$$

← however, it doesnot give exact expression but gives an idea to proceed and then results are found (i.e. here the value of 'a') experimentally.

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12/1/2012 Buckhingham's π- Theorem

If there are 'n' no. of total variables, 'm' no of fundamental quantities then the given system can be grouped into (n-m) π terms.

π — is a dimensionless Group.

X = f(a,b,c,d,e) ('Total no. of Variables = 6)

n → no. of total variable

Que. The resistance force 'F' of a ship is a function of its length (l), velocity (v) and acceleration due to gravity 'g' and fluid properties like density ρ and viscosity μ, Write this relationship in dimensionless form Using Buckhinghams π theorem.

Ans. $F = \phi(L, v, g, \rho, \mu)$

$n = 6$, $m = 3$

$F \rightarrow MLT^{-2}$

$L \rightarrow L$

$V \rightarrow LT^{-1}$

$g \rightarrow LT^{-2}$

$\rho \rightarrow ML^{-3}$

$\mu \rightarrow ML^{-1}T^{-1}$

The given system can be grouped into 6-3 = 3 π terms

Rules for selection of Repeated Variables

- Repeated Variables must be Selected from independent Variables
- no. of repeated Variables = no. of fundamental quantities
- All repeated Variables must have their own dimensions.
  (η & Re donot have dimensions so cannot be taken as repeated variables)

0 Repeated Variable group must Contain all fundamental quantities.

0 More fundamental quantity must be selected.

i.e. in case of selection of μ and g
'g' is given preference as μ came afterwards and is a derived quantity.

If V not give N(r.p.m) $- T^{-1}$ is Considered

In case of 'V' and 'g' 'V' is given preference over 'g' as acceleration is derived from velocity

& 'L' and 'D' में से किसी को भी चुन सकते हैं।

In Fluid mechanics we mostly choose (L, V, ρ)

$$\Rightarrow \pi_1 = F\left[L^{a_1} V^{b_1} \rho^{c_1}\right]$$

$$\pi_2 = g\left[L^{a_2} V^{b_2} \rho^{c_2}\right]$$

$$\pi_3 = \mu\left[L^{a_3} V^{b_3} \rho^{c_3}\right]$$

$$\pi_1 = F.\left[L^{a_1} V^{b_1} \rho^{c_1}\right]$$

$$M^0L^0T^0 = MLT^{-2}\left[L^{a_1}\right]\left[V^{b_1}\right]\left[\rho^{c_1}\right]$$

$$\Rightarrow M^0L^0T^0 = MLT^{-2}\left[L^{a_1}\right]\left[LT^{-1}\right]^{b_1}\left[ML^{-3}\right]^{c_1}$$

$$M^0L^0T^0 = (M^{1+c_1})\ L^{1+a_1+b_1-3c_1}\ T^{-2-b_1}$$

$$1+c_1 = 0 \Rightarrow c_1 = -1.$$

$$-2-b_1 = 0 \Rightarrow b_1 = -2$$

$$1+a_1+b_1-3c_1 = 0$$

$$\Rightarrow 1+a_1-2+3 = 0.$$

$$\Rightarrow a_1 = -2$$

$$\Rightarrow \pi_1 = F\left[L^{-2}V^{-2}\rho^{-1}\right] \Rightarrow \pi_1 = \frac{F}{\rho L^2 V^2}$$

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Similarly,

$$\pi_2 = \frac{gL}{V^2}$$

$$\pi_3 = \frac{\mu}{eVL}$$

$$\pi_1 = \frac{F}{eL^2V^2}$$

as F i.e. dependent Variable is in π_1 (Which is reqd)

$$\therefore \quad \pi_1 = \phi(\pi_2, \pi_3)$$

$$F = eV^2L^2 \, \phi\left(\frac{gL}{V^2}, \frac{\mu}{eVL}\right).$$

Blindrule → If the above expression is asked, take the denominator as repeated Variable (here, e, v, L).

Q113. The pressure generated by pump P is a fx^n of impeller diameter 'D', speed 'N', discharge 'Q', and the fluid properties like μ and e. Then ~~Write~~ Write the relationship in dimensionless form using Buckhingham's π-theorem.

Ans $P = \phi(D, N, Q, e, \mu)$

$n \rightarrow 6 \quad , \quad m = 3$

$Q = L^3T^{-1} \qquad P = ML^{-1}T^{-2}$

$D = L \qquad e = ML^{-3}$

$N = T^{-1} \qquad \mu = ML^{-1}T^{-1}$

$\therefore$ no. of repeated Variables = 6-3 = 3

The 3 repeated Variables are D, N, e

$\pi_1 = P[D^{a_1}N^{b_1}e^{c_1}]$

$\pi_2 = \theta[D^{a_2}N^{b_2}e^{c_2}]$

$\pi_3 = \mu[D^{a_3}N^{b_3}e^{c_3}]$

$\Rightarrow M^0L^0T^0 = ML^{-1}T^{-2}[D^{a_1}N^{b_1}e^{c_1}]$
$= ML^{-1}T^{-2}[L^{a}][T^{-1}]^{b_1}[ML^{-3}]^{c_1}$

$\Rightarrow \pi_1 = \frac{P}{eD^3N^2}$

$M^0L^0T^0 = L^3T^{-1}[L^{a_2}][T^{-1}]^{b_2}[ML^{-3}]^{c_2}$

$\Rightarrow \pi_2 = \frac{\theta}{D^3N}$

$M^0L^0T^0 = ML^{-1}T^{-1}[L^{a_3}][T^{-1}]^{b_3}[ML^{-3}]^{c_3}$

$\pi_3 = \frac{\mu}{eND^2}$

$\Rightarrow \pi_1 = \phi(\pi_2, \pi_3)$

$\Rightarrow \frac{P}{eD^3N^2} = \phi\left(\frac{\theta}{D^3N}, \frac{\mu}{eND^2}\right)$

$\Rightarrow P = eD^3N^2\,\phi\left(\frac{\theta}{D^3N}, \frac{\mu}{eND^2}\right)$

Q114. The discharge 'θ' over a hydraulic structure is found to depend on head 'h', height 'p', acceleration due to gravity 'g', width 'L' and fluid properties like density 'e', viscosity 'μ'.

and surface tension 'σ'. Express this relationship in dimensionless form Using Buckingham's π-theorem.

Ans. $\theta = f(H, P, g, L, e, \mu, \sigma)$.

'V' is not given so incorporating 'g' for Time (T)
e → for mass,
Among H, P, and L we can choose any one of them.

$n = 8$
$m = 3$ (i.e.) (H, g, e)

$H \to L$, $L \to L$, $\sigma \to MT^{-2}$
$P \to L$, $e - ML^{-3}$, $g \to LT^{-2}$
$\theta \to L^3T^{-1}$, $\mu - ML^{-1}T^{-1}$

$$\Rightarrow \pi_1 = \theta [H]^{a_1} [g]^{b_1} [e]^{c_1}$$

$$\Rightarrow \pi_1 = L^3T^{-1} [L]^{a_1} [LT^{-2}]^{b_1} [ML^{-3}]^{c_1}$$

$$\Rightarrow M^0L^0T^0 = L^3T^{-1} [L]^{a_1} [LT^{-2}]^{b_1} [ML^{-3}]^{c_1}$$

⇒ ~~$M^{(1+c_1)} . L^{(1-3+a_1+b_1-3c_1)} . T^{(-2b_1)}$~~

$$M^{c_1} . L^{(3+a_1+b_1-3c_1)} . T^{(-1-2b_1)} = M^0L^0T^0$$

$\Rightarrow c_1 = 0$
$b_1 = -1/2$
$3 + a_1 + b_1 - 3c_1 = 0$
$\Rightarrow 3 + a_1 - 1/2 = 0$
$\Rightarrow a_1 = -5/2$

$$\Rightarrow \pi_1 = \left(\frac{\theta}{\sqrt{g} \times H^{5/2}}\right)$$

$$\pi_2 = \frac{P}{H}$$

$$\pi_3 = \frac{L}{H}$$

$$\pi_4 = \frac{\mu}{\rho g^{1/2} H^{3/2}}$$

$$\pi_5 = \frac{\sigma}{\rho g H^2}$$

$$\pi_1 = \phi(\pi_2, \pi_3, \pi_4, \pi_5)$$

$$\frac{\theta}{g^{1/2} H^{5/2}} = \phi\left(\frac{P}{H}, \frac{L}{H}, \frac{\mu}{\rho g^{1/2} H^{3/2}}, \frac{\sigma}{\rho g H^2}\right)$$

$$\Rightarrow \quad \theta = g^{1/2} H^{5/2} \, \phi\left(\frac{P}{H}, \frac{L}{H}, \frac{\mu}{\rho g^{1/2} H^{3/2}}, \frac{\sigma}{\rho g H^2}\right)$$

SIMILITUDE AND MODELLING

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Various Forces

o Inertia force (Fi) = ma

$$\Rightarrow F_i = \rho l^3 \left(\frac{V}{t}\right)$$

$$\Rightarrow F_i = \rho l^2 \times \frac{\boxed{L} \times V}{T} = \rho l^2 V^2$$ {As ρ, V, L is imp- in fluid mechanics}

$$\Rightarrow F_i = \rho l^2 V^2$$

o Viscous Force (Fv)

$$F_V = \frac{\mu A V}{y}$$

$$\Rightarrow F_V = \frac{\mu L^2 V}{L} \Rightarrow F_V = \mu V L$$

o Gravity Force (Fg)

$$F_g = mg$$
$$= \rho l^3 g$$

o Surface Tension force (Fs)

$$\sigma = \frac{F}{L} \Rightarrow F_s = \sigma \times L$$

o Pressure force (Fp)

$$P = \frac{F_P}{A} \Rightarrow F_P = P \times A$$
$$\Rightarrow F_P = P L^2$$

o Elastic Force (FE)

When a fluid is Compressed there is a rise in pressure, this rise in pressure is proportional to Bulk Modulus 'k'. This rise in pressure gives rise to a force known as elastic force

$$\Rightarrow F_E = K l^2$$ (where K is Bulk-Modulus)

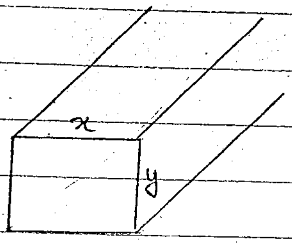
Various Dimensionless Number in Fluid Mechanics

Reynold's Number

It is the ratio of Inertia force to viscous force.

$$\Rightarrow Re = \frac{F_i}{F_v} = \frac{\rho L^2 V^2}{\mu V L} = \frac{\rho V L}{\mu}$$

and Reynold's no. plays a significant role where viscous forces are dominating.



$$Re = \frac{\rho V D_{eq}}{\mu}$$

$D_{eq} \rightarrow$ Circle dia. equivalent $= \frac{4A}{P}$

$$= \frac{4 \times xy}{2(x+y)} = \frac{2xy}{(x+y)}$$

Euler Number

It is the ratio of Inertia force to pressure force

$$Eu = \frac{F_i}{F_P} = \frac{\rho L^2 V^2}{P L^2} = \frac{\rho V^2}{P}$$

here pressure force dominates.

Weber Number

It is the ratio of Inertia force to surface tension force.

$$We = \frac{F_i}{F_s} = \frac{\rho L^2 V^2}{\sigma L} = \frac{\rho L V^2}{\sigma}$$

here surface tension dominates

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- Froude No (F_r)

It is the ratio of inertia force and gravity force. This no. is used in analysis of spillways, towing of ships, river.

$$F_r = \frac{F_i}{F_g} = \frac{\rho L^2 V^2}{\rho L^3 g} = \frac{V^2}{gL}$$

$$\Rightarrow F_r = \frac{V^2}{gL}$$

Here gravity force dominates.

- Mach Number (Ma)

It is the ratio of square root of Inertia force to elastic force,

$$\Rightarrow Ma = \sqrt{\frac{F_i}{F_e}} = \sqrt{\frac{\rho L^2 V^2}{K L^2}} = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C}$$

$$Ma = \frac{V}{C} \quad \text{Where } C = \sqrt{\frac{K}{\rho}} \text{ (velocity of sound)}$$

Whenever we come across ~~elas~~ bulk modulus or elastic force we use mach Number

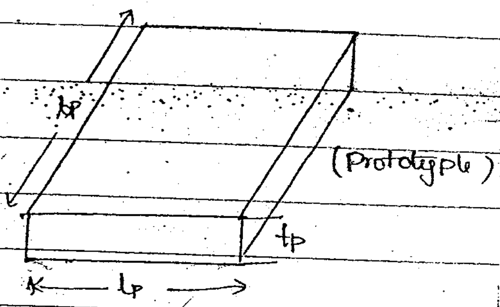
$$\text{Cauchy Number} = \frac{F_i}{F_e}$$

Geometric Similarity

Model and prototype are said to be in Geometric similarity when ratio of dimensions in model and prototype is same

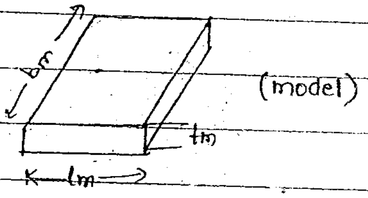
$$\frac{l_m}{l_p} = \frac{b_m}{b_p} = \frac{t_m}{t_p} = l_r \text{ (length ratio)}$$

$$\text{(Area ratio) } A_r = \frac{A_m}{A_p} = \frac{b_m l_m}{b_p l_p} = l_r l_r = l_r^2.$$



$$\text{Vol. ratio} = \frac{l_m b_m t_m}{l_p b_p t_p} = (l_r)^3$$

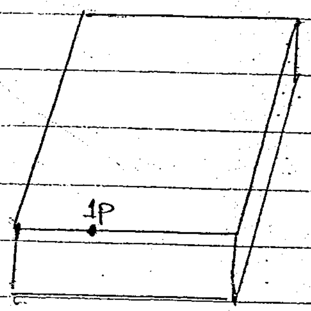
model ~~or~~ may be smaller or bigger than the prototype.



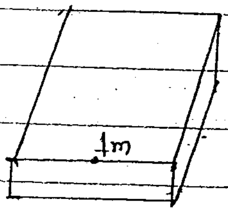
Kinematic Similarity

Model and prototype are said to be in Kinematic Similarity if ratio of velocity, acceleration at corresponding points in model and prototype is same. For Kinematic Similarity, geometric similarity is must.

Dynamic Similarity



Prototype



model

Model and prototype are said to be in dynamic similarity if ratio of forces in model and prototype at corr. points are same.

Various dimensionless numbers are equated for maintaining dynamic similarity.

$$\frac{(F_i)_{1p}}{(F_i)_{1m}} = \frac{(F_v)_{1p}}{(F_v)_{1m}}$$ {When viscous force is dominating}.

$$\Rightarrow \frac{(F_i)_{1p}}{(F_v)_{1p}} = \frac{(F_i)_{1m}}{(F_v)_{1m}} \Rightarrow (Re)_{1p} = (Re)_{1m}$$ (Reynold's model law)

Q116. A fluid flow phenomenon is to be studied, which is to be constructed by using Reynold's model law. Find the ratio of model to prototype expressions for velocity and discharge in terms of kinematic viscosity.

Ans. It's Reynold's model law,

$$\Rightarrow (Re)_m = (Re)_p$$

$$\Rightarrow Re = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$$

$$\Rightarrow Re = \frac{VL}{\nu}$$

$$(Re)_m = (Re)_p$$

$$\Rightarrow \left(\frac{VL}{\nu}\right)_m = \left(\frac{VL}{\nu}\right)_p$$

$$\Rightarrow \frac{V_m L_m}{\nu_m} = \frac{L_p V_p}{\nu_p}$$

$$\Rightarrow \frac{V_m}{V_p} = \frac{L_p}{L_m} \cdot \frac{\nu_m}{\nu_p}$$

$$\Rightarrow \frac{V_m}{V_p} = V_r = \frac{1}{L_r} \times \nu_r$$

$$\Rightarrow V_r = \frac{\nu_r}{L_r}$$

$$\Rightarrow Q_r = \frac{Q_m}{Q_p} = \frac{A_m V_m}{A_p V_p} = L_r^2 \times V_r$$

$$\Rightarrow Q_r = L_r^2 \times \frac{\nu_r}{L_r} = (\nu_r L_r)$$

Q116 For Froude Model law, find the ratio of Velocity and discharge

Ans. $F_r = \frac{V^2}{gL}$

$$(F_r)_m = (F_r)_p \Rightarrow \left(\frac{V^2}{gL}\right)_m = \left(\frac{V^2}{gL}\right)_p$$

$$\Rightarrow \frac{V_m^2}{L_m} = \frac{V_p^2}{L_p}$$

$$\Rightarrow \frac{V_m^2}{V_p^2} = \frac{L_m}{L_p} \Rightarrow \frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} = \sqrt{L_r}$$

$$Q = AV$$

$$\Rightarrow Q_r = \frac{Q_m}{Q_p} = \frac{A_m V_m}{A_p V_p} = L_r^2 \times \sqrt{L_r}$$

$$\Rightarrow Q = L_r^{5/2}$$

Q117 Oil of density 917kg/m³ and viscosity 0.29 Ns/m² flows in a pipe of 15cm diameter at a velocity of 2m/s. What would be the Velocity of flow of water in a 1cm diameter pipe to make two flows dynamically similar, $\rho_{water} = 998 kg/m^3$, $\mu = 1.31\times10^{-3} \frac{N\text{-}S}{m^2}$

Ans.

Oil	Water
$\rho = 917$	$\rho = 998$
$\mu = 0.29$	$\mu = 1.31 \times 10^{-3}$
$D = 15cm$	$D = 1cm$
$V = 2m/s$	$V = ?$

$Re_{oil} = Re_{H_2O}$

$$\Rightarrow \left(\frac{\rho V D}{\mu}\right)_{oil} = \left(\frac{\rho V D}{\mu}\right)_{water}$$

$$\Rightarrow \frac{917 \times 2 \times 15}{0.29} = \frac{998 \times V \times 1}{1.31 \times 10^{-3}}$$

$$\Rightarrow V = 0.1245 m/s$$

Q118. Obtain an expression for scale ratio of model to prototype which has to satisfy both Reynold's and froude's model law. Express your answer in terms of kinematic viscosity.

Ans. $Re = \frac{\rho V l}{\mu} \Rightarrow Re = \frac{VL}{\nu}$

$Re_m = Re_p$

$$\Rightarrow \left(\frac{VL}{\nu}\right)_m = \left(\frac{VL}{\nu}\right)_p \Rightarrow \frac{V_m L_m}{\nu_m} = \frac{V_p L_p}{\nu_p}$$

$$\Rightarrow \frac{V_m}{V_p} = \frac{L_p}{L_m} \times \frac{\nu_m}{\nu_p} = \frac{1}{L_r} \cdot \nu_r \quad \text{——} ①$$

$\therefore (F_r)_m = (F_r)_p$

$\Rightarrow \left(\frac{V^2}{gL}\right)_m = \left(\frac{V^2}{gL}\right)_p$

$\Rightarrow \frac{V_m^2}{L_m} = \frac{V_p^2}{L_p}$

$\Rightarrow \frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} \Rightarrow \frac{V_m}{V_p} = \sqrt{L_r}$ —— ②

from ① and ②

$\frac{V_r}{L_r} = \sqrt{L_r}$

$\Rightarrow V_r = \sqrt{L_r} \cdot L_r$

$\Rightarrow L_r = V_r^{2/3}$

Q. If a 1m long model of a ship is towed at a speed of 81cm/s in a tank to what speed a ship of 64m long ship is to be towed for dynamic similarity.

Ans.

model	Prototype
$V_m = 81$ cm/s	$V_p = ?$
$L_m = 1$m	$L_p = 64$m

$\Rightarrow \left(\frac{V^2}{gL}\right)_m = \left(\frac{V^2}{gL}\right)_p \Rightarrow \frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} \Rightarrow \frac{81\text{cm/s}}{V_p} = \sqrt{\frac{1}{64}}$

$\Rightarrow V_p = 648$ cm/s

Q120 A spill way of an irrigation project is to be studied by means of a model constructed to a scale of 1:9. The prototype discharge is 1000 m³/sec. Neglecting viscous and surface tension effects find the discharge of model.

Ans. For Froude model law

$$Q_r = (L_r)^{5/2}$$

$$\Rightarrow \frac{Q_m}{Q_p} = \left(\frac{L_m}{L_p}\right)^{5/2}$$

$$\Rightarrow \frac{Q_m}{1000} = \left(\frac{1}{9}\right)^{5/2} \Rightarrow Q_m = 4.11\,m^3/sec$$

Q121. A river model is constructed to a horizontal scale of 1: ~~100~~ 30 and vertical scale of 1:100. The discharge in model is 0.1 m³/s then find the discharge in prototype.

Ans. $x \rightarrow$ horizontal length
$y \rightarrow$ vertical length

$$(\text{Horizontal})_{ratio} = \frac{1}{30}$$

$$(\text{Vertical})_{ratio} = \frac{1}{100}$$

$$A = x \times y$$

$$\Rightarrow A_r = \frac{A_m}{A_p} = \frac{x_m \times y_m}{x_p \times y_p}$$

$$\Rightarrow A_r = (\text{Horizontal})_r \times (\text{Vertical})_r$$

$$\Rightarrow A_r = \frac{1}{30} \times \frac{1}{100} = \frac{1}{3000}$$

$(F_r)_m = (F_r)_p$

⇒ $\left(\frac{V^2}{gL}\right)_m = \left(\frac{V^2}{gL}\right)_p$

⇒ $\frac{V_m^2}{L_m} = \frac{V_p^2}{L_p}$

⇒ $\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}}$

as in Fr.no gravity force dominates and gravity acts in vertical dirxn so Lm & Lp are in vertical dirxns.

⇒ $\frac{V_m}{V_p} = \left(\frac{L_m}{L_p}\right)^{1/2}$

⇒ $\frac{V_m}{V_p} = \left(\frac{1}{100}\right)^{1/2} = \frac{1}{10}$

$Q = AV$

⇒ $Q_H = A_H V_H$

⇒ $Q_H = \frac{A_m}{A_p} \times \frac{V_m}{V_p}$

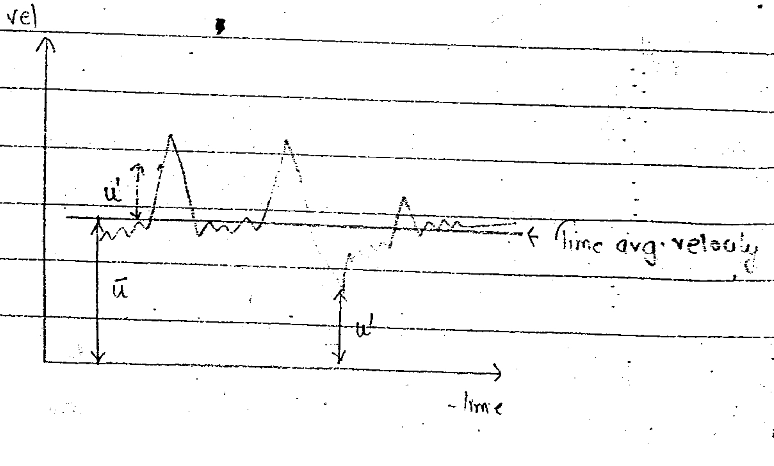
⇒ $\frac{Q_m}{Q_p} = \frac{1}{3000} \times \frac{1}{10}$

⇒ $\frac{0.1}{Q_p} = \frac{1}{30000}$ ⇒ $Q_p = 3000\, m^3/sec$

13/1/2012 Turbulent flow :-

$u = u' + \bar{u}$
↑
fluctuating comp.

$u = \bar{U} \pm U'$



The first advancement in Turbulent flow by

Boussinesq eqn

$$\tau = \underset{\substack{\uparrow \qquad \uparrow \\ \text{eddy viscosity} \quad \text{flow property}}}{\eta} \frac{du}{dy} + \overset{\text{fluid property}}{\mu} \frac{du}{dy}$$

As it is difficult to determine eddy viscosity (η) {neta} so we don't use it

Reynold's Shear Stress Eqn

Reynold gave the eqn of Turbulent Shear Stress as

$$\tau = \rho u'v'$$

Where u' and v' are fluctuating components in x and y dirns respectively.

Prandtl's Mixing Length theory (for Turbulent flow through Pipes)

According to prandtl, mixing length is that length in transverse dirn wherein fluid particles after colliding loose excess momentum same as the local environment. from prandtl's experiment $l = 0.4y$ Where l is prandtl's mixing length and y is distance from pipe wall.

Acc. to prandtl $\boxed{u' = v' = l\frac{du}{dy}}$

$$\tau = \rho u'v'$$

$$= \rho . l . \frac{du}{dy} . l\frac{du}{dy}$$

$$= \boxed{\rho l^2 \left(\frac{du}{dy}\right)^2}$$

Velocity Distribution

$$\frac{\tau}{e} = l^2\left(\frac{du}{dy}\right)^2$$

$$\Rightarrow \sqrt{\frac{\tau}{e}} = l\left(\frac{du}{dy}\right)$$

$$\Rightarrow V^*(\text{Shear stress velocity}) = l\frac{du}{dy}$$

$$\Rightarrow V^* = 0.4y\frac{du}{dy}$$

$$\Rightarrow \frac{V_*}{0.4}\frac{dy}{y} = du$$

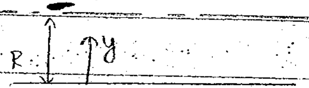
$$\Rightarrow \boxed{2.5V_*\ln y = u + C} \text{ ——— I}$$

In turbulent flow velocity distribution is logarithmic.

u → local velocity at a distance y from pipe wall.

R → Radius of pipe.

At y = R (centre) u = Umax



$$2.5V_*\ln y = u + C$$

$$2.5V_*\ln R = U_{max} + C$$

$$\Rightarrow C = 2.5V_*\ln R - U_{max}$$

$$\Rightarrow 2.5V_*\ln y = U + 2.5V_*\ln R - U_{max}$$

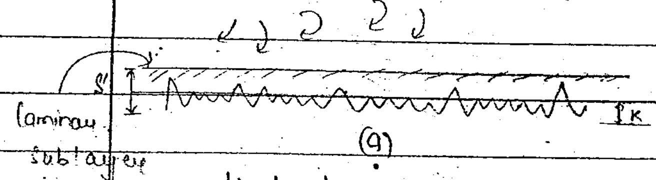
$$\Rightarrow U_{max} - U = 2.5V_*\ln R - 2.5V_*\ln y$$

$$\Rightarrow U_{max} - u = 2.5V_*\ln\frac{R}{y}$$

$$\frac{U_{max} - u}{V^*} = 2.5 \ln \frac{R}{y}$$

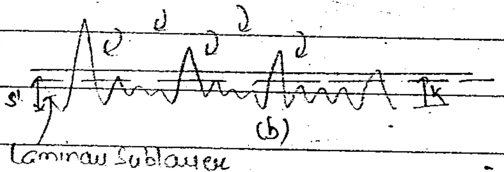
$$\boxed{\frac{U_{max} - u}{V_*} = 5.75 \log_{10}\left(\frac{R}{y}\right)}$$ ← Prandtl's velocity defect eqn. —— (2).

Hydrodynamically Smooth and Hydrodynamically Rough Boundary



(a)

Hydrodynamically Smooth



(b)

Hydrodynamically Rough.

k → Centre line average

δ' → laminar Sublayer

In (a), the boundary Sub-layer acts as a blanket and turbulent flow don't Come in Contact with peaks and valleys whereas in (b) the turbulent flow Comes in Contact with peaks and valleys.

Nikuradse's Condn for Rough and smooth pipes

- $\frac{k}{\delta'} < 0.25$ → Smooth boundary
- $\frac{k}{\delta'} > 6$ → rough boundary
- $0.25 < \frac{k}{\delta'} < 6$ → Transition boundary

Reynold's Condition for Rough and Smooth pipes

- $\frac{V_* K}{\nu} < 4 \rightarrow$ Roug Smooth
- $\frac{V_* K}{\nu} > 100 \rightarrow$ Rough
- $4 < \frac{V_* K}{\nu} < 100 \rightarrow$ Transition.

$\frac{V_* K}{\nu}$ is known as Reynold's Roughness number.

From Nikuradse's eq^n laminar Sub-layer thickness δ' is equal to

$$\underset{\uparrow \text{ laminar sublayer}}{\delta'} = \frac{11.6\,\nu \leftarrow \text{Kinematic viscosity}}{V_* \leftarrow \text{Shear velocity}} \quad \{\text{for pipes}\}$$

$$2.5 V_* \ln y = U + c$$

at $y = 0$, $\ln y$ is Undefined

∴ We assume to take at a distance y' i.e. small distance from the Wall where velocity is almost zero.

$$\text{At } y = y' \Rightarrow u = 0$$

$$2.5 V_* \ln y' = 0 + c$$

$$\Rightarrow c = 2.5 V_* \ln y'$$

$$\Rightarrow 2.5 V_* \ln y = u + 2.5 V_* \ln y'$$

$$\Rightarrow 2.5 V_* [\ln y - \ln y'] = u$$

$$\Rightarrow \boxed{\frac{u}{V_*} = 2.5 \ln\left(\frac{y}{y'}\right)} \leftarrow \text{Valid for both Smooth and Rough pipes.}$$

From experiments it is found that

$y' = \frac{\delta'}{107}$ → Smooth boundaries

$y' = \frac{K}{30}$ → Rough boundaries.

Velocity distribution for Smooth pipes

$$\frac{u}{V_*} = 2.5 \ln\left[\frac{y}{y'}\right)$$

$$\Rightarrow \frac{u}{V^*} = 2.5 \ln\left[\frac{y}{\delta'/107}\right]$$

$$\Rightarrow \frac{u}{V_*} = 2.5 \ln\left[\frac{107y}{\delta'}\right]$$

$$\Rightarrow \frac{u}{V_*} = 2.5 \ln\left[\frac{107y}{\frac{11.6\nu}{V_*}}\right]$$

$$\Rightarrow \frac{u}{V_*} = 2.5 \ln\left[\left(\frac{107}{11.6}\right)\left(\frac{V_* y}{\nu}\right)\right]$$

$$\Rightarrow \frac{u}{V_*} = 5.75 \log_{10}\left[\left(\frac{107}{11.6}\right)\right] + 5.75 \log_{10}\left[\frac{V_* y}{\nu}\right]$$

$$\Rightarrow \boxed{\frac{u}{V_*} = 5.75 \log_{10}\left(\frac{V_* y}{\nu}\right) + 5.5}$$ ← Valid for smooth pipes.

Velocity distribution for Rough pipes

$$\frac{u}{V_*} = 2.5 \ln\left(\frac{y}{y'}\right)$$

$$\frac{u}{V_*} = 5.75 \log_{10}\left(\frac{y}{y'}\right)$$

for rough pipes $y' = \frac{K}{30}$

$$\frac{u}{V_*} = 5.75 \log_{10}\left[\frac{y}{K/30}\right]$$

$$\frac{u}{V_*} = 5.75 \log_{10}\left[\frac{30y}{K}\right]$$

$$\Rightarrow \frac{u}{V_*} = 5.75 \log_{10} 30 + 5.75 \log_{10}\left(\frac{y}{K}\right)$$

$$\Rightarrow \boxed{\frac{u}{V_*} = 5.75 \log_{10} \frac{y}{K} + 8.5} \leftarrow \text{Valid for Rough pipes only}$$

Average velocity

$$\text{Avg. Velocity} = \frac{Q}{A} = \frac{\int u\,dA}{\int dA}$$

Avg. velocity

$$\Rightarrow \frac{V}{V_*} = 5.75 \log_{10}\left(\frac{V_* R}{\nu}\right) + 1.75 \leftarrow \text{Smooth}$$

Shear velocity

$$\frac{V}{V_*} = 5.75 \log_{10}\left(\frac{R}{K}\right) + 4.75 \rightarrow \text{Rough}$$

$$\frac{u}{V_*} = 5.75 \log_{10}\left(\frac{V_* y}{\nu}\right) + 5.5 \leftarrow \text{Smooth}$$

$$\frac{u}{V_*} = 5.75 \log_{10}\left(\frac{y}{K}\right) + 8.5 \leftarrow \text{Rough}$$

$$\boxed{\frac{U-V}{V_*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75}$$ ← Valid for both rough and smooth pipe ****

Friction factor in Turbulent Flow

Smooth Pipes	Rough Pipes
$f = \frac{0.3164}{(Re)^{1/4}}$ (upto $Re = 10^5$)	$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{R}{k}\right) + 1.74$
$f = 0.0032 + \frac{0.221}{(Re)^{0.232}}$ $Re = 10^5$ to 4×10^7	-

** The friction factor depends on Reynold's no. and average height of Roughness. In case of smooth pipes friction factor depends on Reynold's no only

Whereas in Rough pipes ~~friction~~ friction depends on average height 'k'.

And for laminar flow friction factor depends on Reynold's no. only.

Q122. Find the distance from the pipe wall by which the local velocity is equal to average velocity in turbulent.

Ans. $\frac{U-V}{V_*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$

$U = V$ (given)

$\Rightarrow \quad 0 = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$

$\Rightarrow \quad 5.75 \log_{10} \frac{y}{R} = -3.75$

$\Rightarrow \quad \frac{-3.75}{5.75} \log_{10} \frac{y}{R}$

$\Rightarrow \quad \frac{y}{R} = 10^{-3.75/5.75}$

$\Rightarrow y/R = 0.223 \quad \Rightarrow y = \underline{0.223R}$

Q123. Show that for turbulent flow in a pipe, the ratio of maximum velocity to avg. velocity is given by

$$\frac{U_{max}}{V} = 1 + 1.33\sqrt{f}$$

Ans. $\frac{U-V}{V_*} = 5.75 \log_{10}\left(\frac{y}{R}\right) + 3.75$

at the centre, $y = R \Rightarrow U = U_{max}$

$\Rightarrow \frac{U_{max} - V}{V_*} = 5.75 \log_{10}(1) + 3.75$

$\Rightarrow \frac{U_{max} - V}{V_*} = 3.75$

$\Rightarrow U_{max} - V = 3.75 V_* \quad$ —— ①

and $\tau_0 = \frac{\rho f V^2}{8}$ (τ_0 is wall shear stress, valid for both laminar and turbulent flow as in both case there is a layer of laminar flow just above wall).

$\Rightarrow \frac{\tau_0}{\rho} = \frac{f}{8} V^2 \Rightarrow \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{f}{8}} \cdot V$

$V_* = \sqrt{\frac{f}{8}} \cdot V \quad$ —— ②

Substitute ② in ①

$$U_{max} - V = 3.75 \sqrt{\frac{f}{8}} \times V$$

$$\Rightarrow \frac{U_{max} - V}{V} = 3.75 \sqrt{\frac{f}{8}}$$

$$\Rightarrow \frac{U_{max}}{V} - 1 = 1.33\sqrt{f}$$

$$\Rightarrow \frac{U_{max}}{V} = 1 + 1.33\sqrt{f}$$

Q124. A pipe carrying water has average height of Roughness (K) = 0.48mm. The diameter of the pipe is 0.6m. Length is equal to 4500 mm. The discharge of water is 0.6 m³/sec find the power required to maintain the flow. Take viscosity as 1 centi poise (10^{-3} NS/m²). Assume the pipe to be rough.

Ans.

$$Re = \frac{\rho V D}{\mu} = \frac{10^3 \times V \times 0.6}{10^{-3}}$$

$$Q = AV$$

$$Q = \frac{\pi}{4} D^2 V$$

$$\Rightarrow V = \frac{4Q}{\pi D^2} = \frac{4 \times 0.6}{\pi \times (0.6)^2} \Rightarrow V = 2.12 \text{ m/s}$$

$$Re = \frac{10^3 \times 2.12 \times 0.16}{10^{-3}} = 1.2 \times 10^6 \rightarrow \text{Turbulent}$$

$$\therefore \frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{R}{K}\right) + 1.74$$

$$\Rightarrow \frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{0.3}{0.48 \times 10^{-3}}\right) + 1.74$$

$f = 0.0186$

$h_L = \frac{fLV^2}{2gD} \Rightarrow h_L = \frac{0.018 \times 4.5 \times 2.12^2}{2 \times 9.81 \times 0.6} = 0.032\,m$

$P = WQh_L$

$P = 9810 \times 0.6 \times 0.032$

$\Rightarrow P = \underline{\underline{188.35\,W}}$

Q125 — A pipe carrying water has average height of roughness of 0.01mm What type of boundary is this, the shear stress developed is $4.9\,N/m^2$ and viscosity $= 0.001\,\frac{N\text{-}s}{m^2}$. Use Reynold's Condn.

Ans.

$K = 0.01\,mm$

$\tau = 4.9$

$\mu = 0.001$

$\rho = 10^3$ (Water)

$\frac{V_* K}{\nu} = ?$

as $V_* = \sqrt{\frac{\tau}{\rho}} = \sqrt{\frac{4.9}{10^3}} = 0.07\,m/s$

$\nu = \frac{\mu}{\rho} = \frac{0.001}{10^3} = 10^{-6}$

Reynold's roughness no. $= \frac{0.07 \times 0.01 \times 10^{-3}}{10^{-6}}$

$= 0.7 < 4 \rightarrow$ Smooth boundary.

Q126. A rough pipe of 0.1m diameter carries water at the rate of 50lt/s the average height of roughness is 0.15mm. find (a). friction factor

(b). Shear Stress at pipe surface

(c). Shear velocity

(d). Maximum velocity

$\nu = 10^{-6} \frac{m^2}{s}$, $\rho = 1000 kg/m^3$.

Ans.

$d = 0.1m$

$Q = 5 litre/sec$

$K = 0.15mm$

$\nu = 10^{-6}$, $\rho = 10^3$

$Q = AV$.

$\Rightarrow 5 \times 10^{-3} = \frac{\pi}{4}(0.1)^2 \times V$

$\Rightarrow V = 6.36$ m/s

Re = ________ flow is turbulent.

∴ for Rough pipe

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{R}{k}\right) + 1.74$$

$$\frac{1}{\sqrt{f}} = 2\log_{10}\left(\frac{0.05}{0.15 \times 10^{-3}}\right) + 1.74$$

$$f = 0.0216$$

$$\frac{U_{max}}{V} = 1 + 1.33\sqrt{f}$$

$$\Rightarrow \frac{U_{max}}{V} = 1 + 1.33\sqrt{0.0216}$$

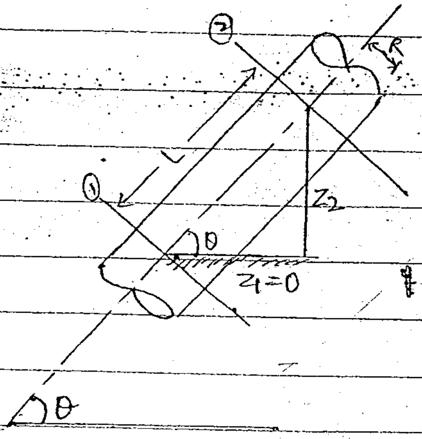
$$\Rightarrow U_{max} = 7.61 m/s$$

$$\Rightarrow \tau_0 = \frac{\rho f V^2}{8} = \frac{10^3 \times 0.0216 \times 6.36}{8}$$

⇒ $\tau_0 = 109.8\ N/m^2$

$V_* = \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{109.8}{10^3}} = 0.331\ m/s$

*** Darcy- Weisbach Eqn



$Q = A_1V_1 = A_2V_2$

as $A_1 = A_2$

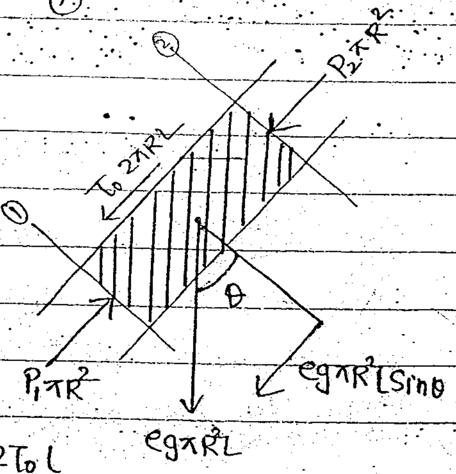
⇒ $V_1 = V_2$

⇒ $\frac{P_1}{w} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{w} + \frac{V_2^2}{2g} + z_2 + h_L$

$\frac{P_1}{w} + 0 = \frac{P_2}{w} + l\sin\theta + h_L$

⇒ $\frac{P_1}{w} - \frac{P_2}{w} - l\sin\theta = h_L$ —— ①

now,



$P_1\pi R^2 - P_2\pi R^2 - \rho g\pi R^2 l\sin\theta - \tau_0 2\pi RL = 0$

⇒ $P_1R - P_2R - \rho gR\, l\sin\theta = 2\tau_0 l$

$\frac{(P_1-P_2)\cancel{R}}{\rho g\cancel{R}} - \frac{\cancel{\rho}\, l\sin\theta\times\cancel{R}\times\cancel{g}}{\cancel{\rho g R}} = \frac{2\tau_0 l}{\rho gR}$

⇒ $\frac{P_1-P_2}{w} - l\sin\theta = \frac{2\tau_0 L}{\rho gR}$ —— ②

from ① and ②

$h_L = \frac{2\tau_0 L}{\rho gR}$ ⇒ $h_L = \frac{2L}{\rho gR} \times \frac{\rho f V^2}{8}$

⇒ $h = \frac{fLV^2}{4gR}$ ⇒ $\boxed{h_L = \frac{fLV^2}{2gD}}$ Imp for 2012